

# Kinematics and dynamics of early-type galaxies

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AIP

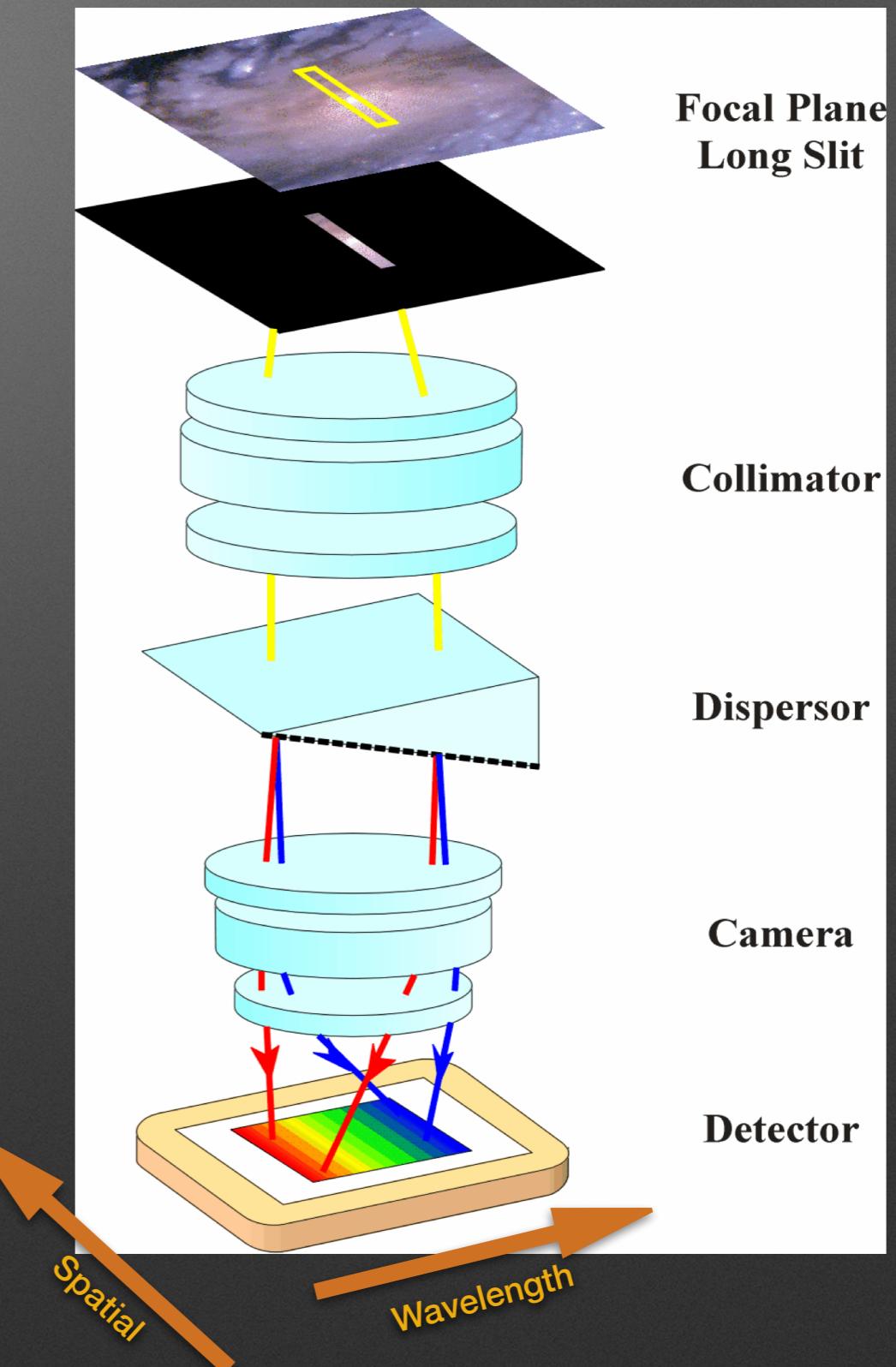
# Outline

- obtaining spectra
- extracting kinematics
- stellar kinematics in early-type galaxies
- dynamics of galaxies
- dynamical models

# Obtaining spectra

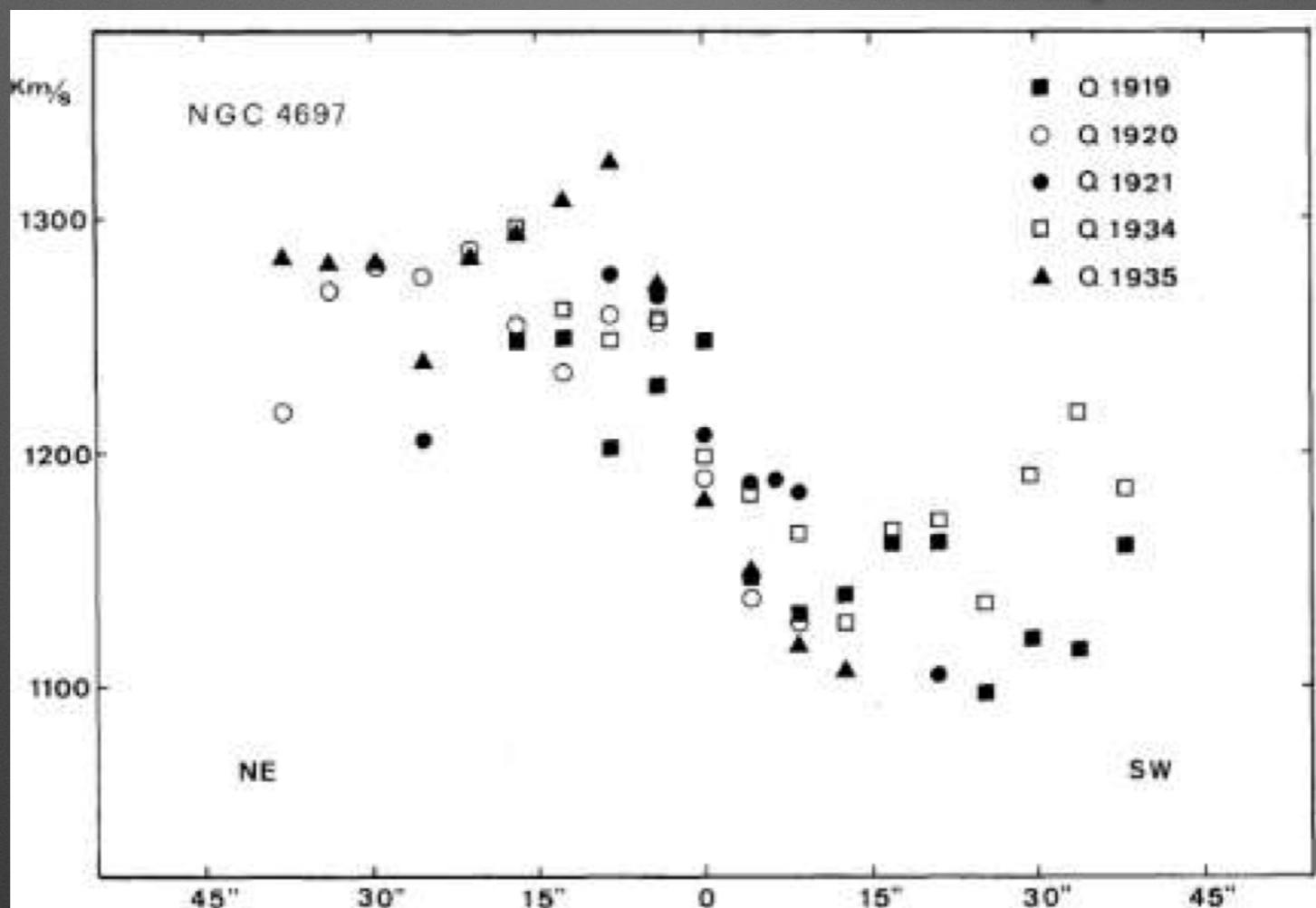
# Instruments of choice (and necessity)

- ancient history (1970)
- Long Slit: work horse of galaxy kinematics (1980s - 1990s)-
- present day: IFU + (multiple) long-slits
- Future: IFUs (?)
- always use the best suited instrument/model
- basic principle the same
  - aperture, dispersor, detector (+ loads of optics)



# First rotation curve(s)

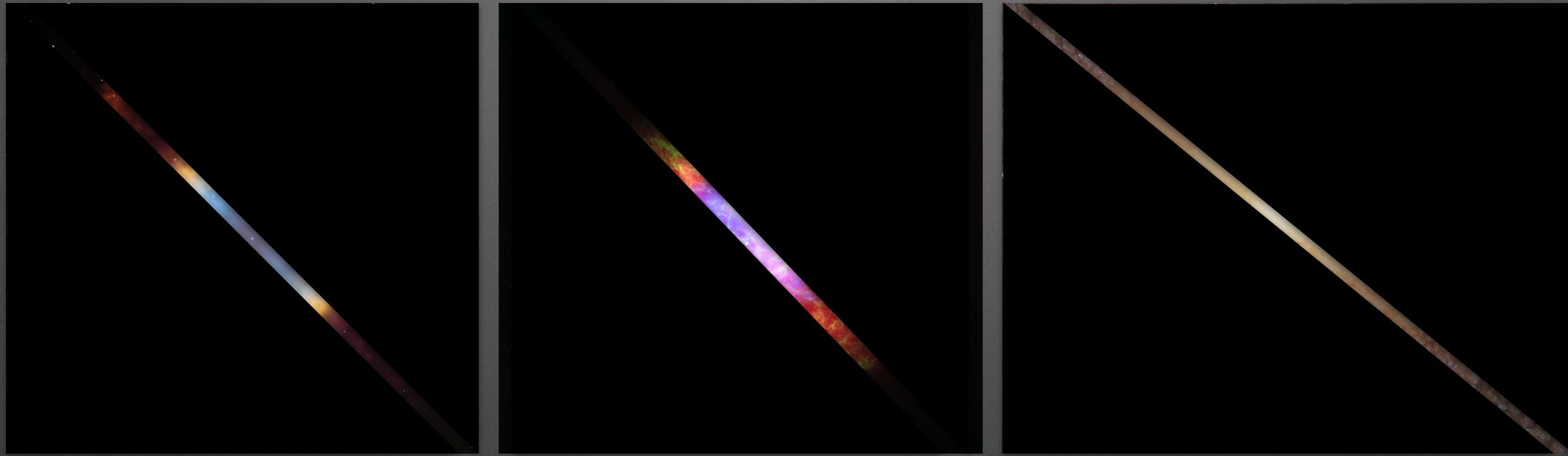
Bertola & Capaccioli 1975



- NGC4697 (elliptical)
- Palomar 5m: 2h observations
- angular momentum lower than for spirals (5-30 times)

# Objects are not 1D

(Except at high z)



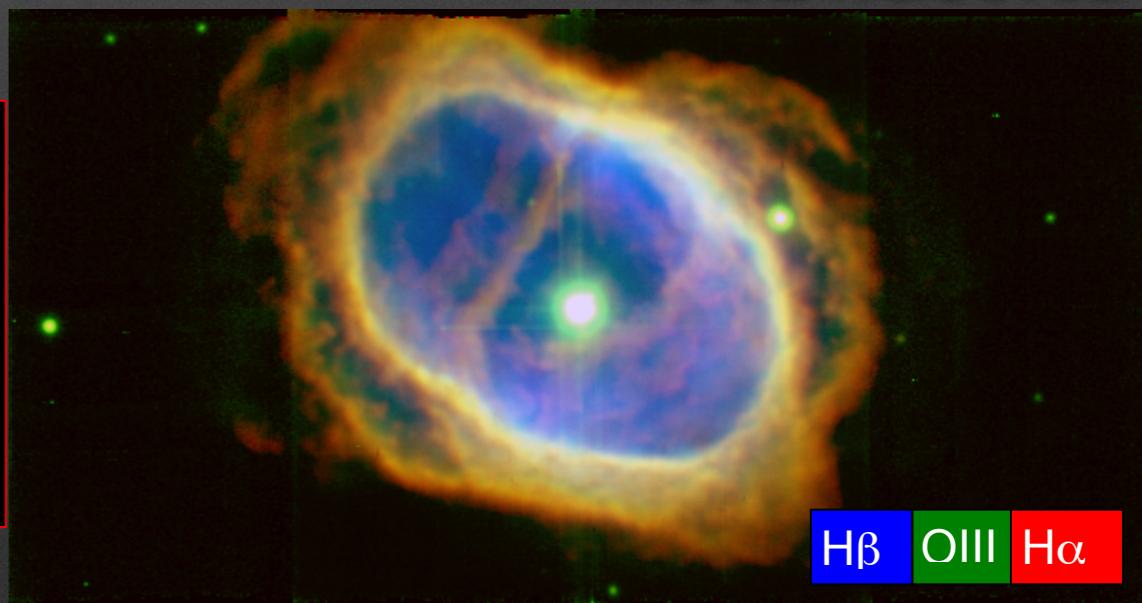
MUSE in Lyon



MUSE Jupiter (OH line)

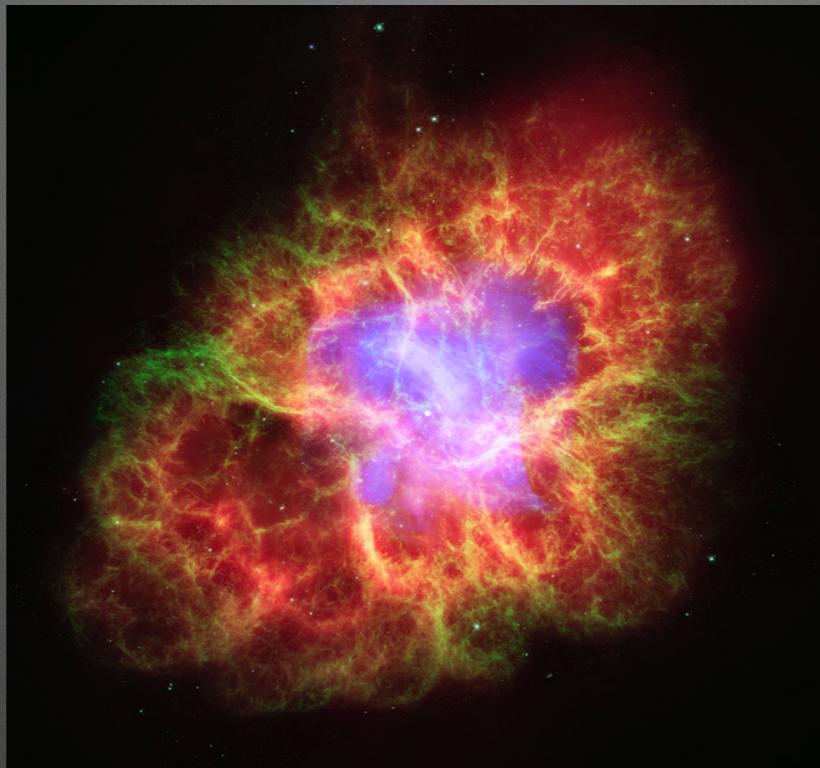


MUSE NGC3132



# Objects are not 1D

(Except at high z)



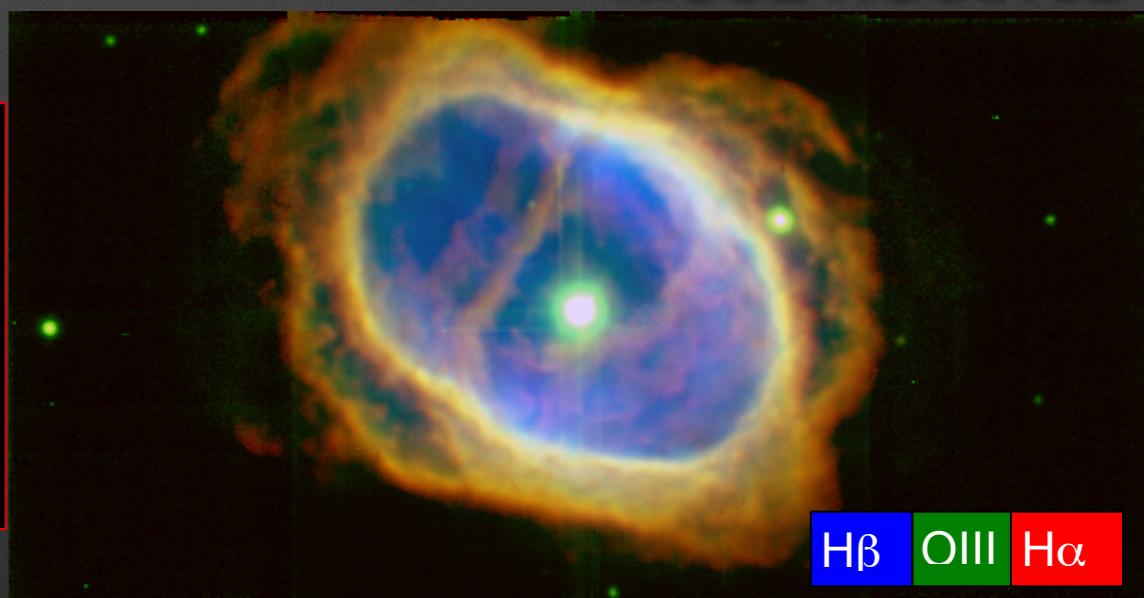
MUSE in Lyon



MUSE Jupiter (OH line)

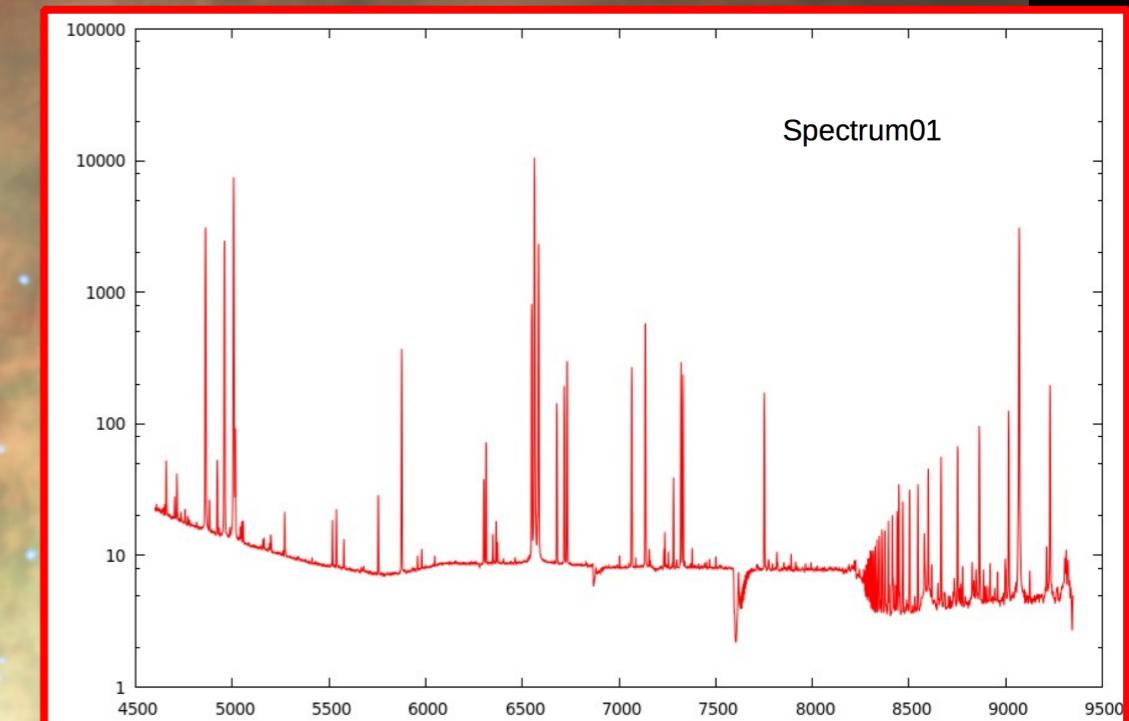
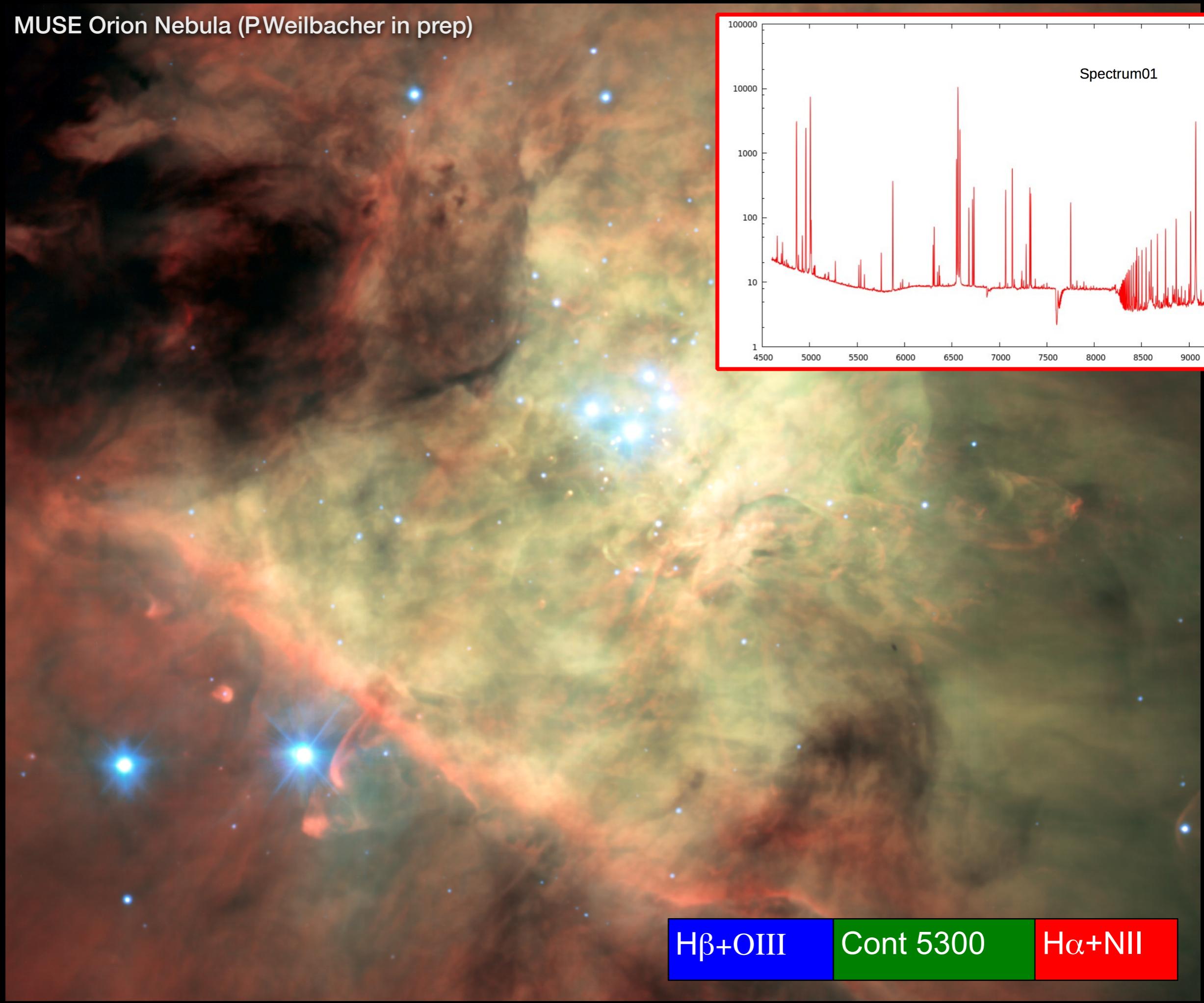


MUSE NGC3132



H $\beta$  OIII H $\alpha$

MUSE Orion Nebula (P.Weilbacher in prep)

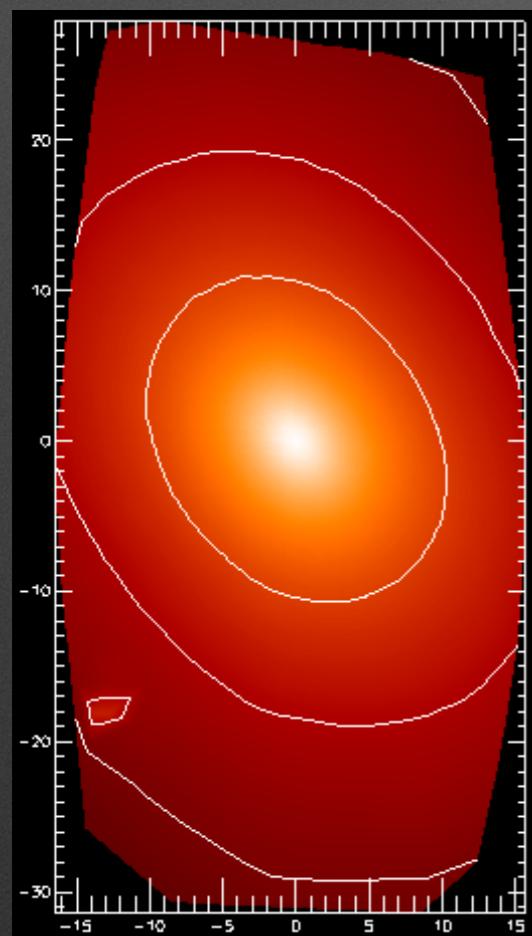


H $\beta$ +OIII

Cont 5300

H $\alpha$ +NII

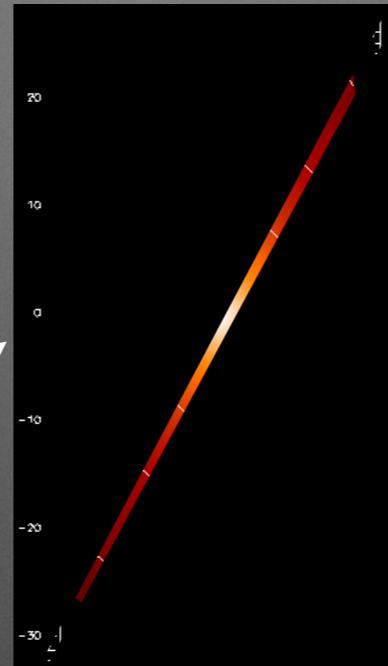
# Long-slit vs IFU



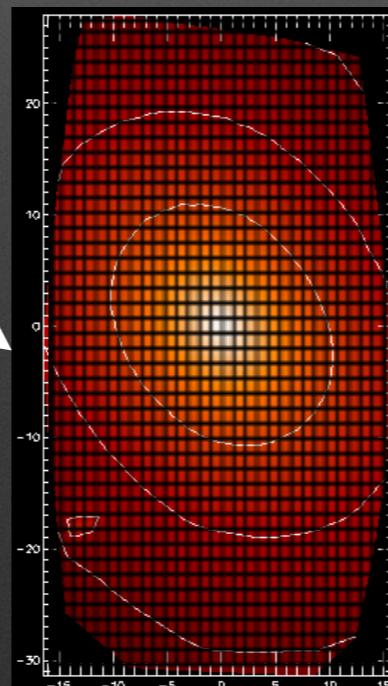
Galaxy Image

Long  
slit

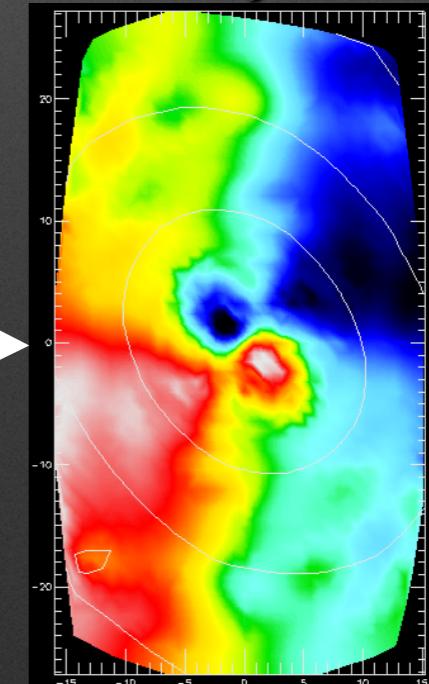
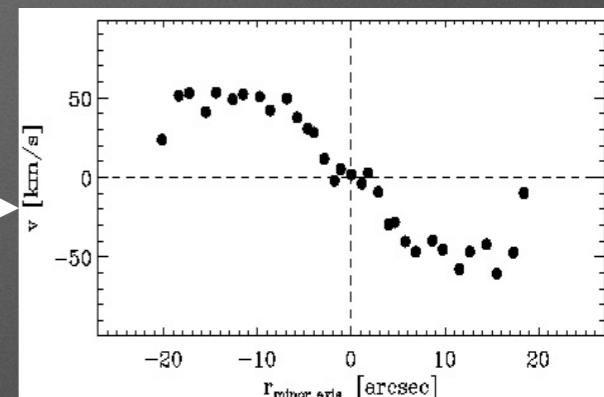
Integral  
field



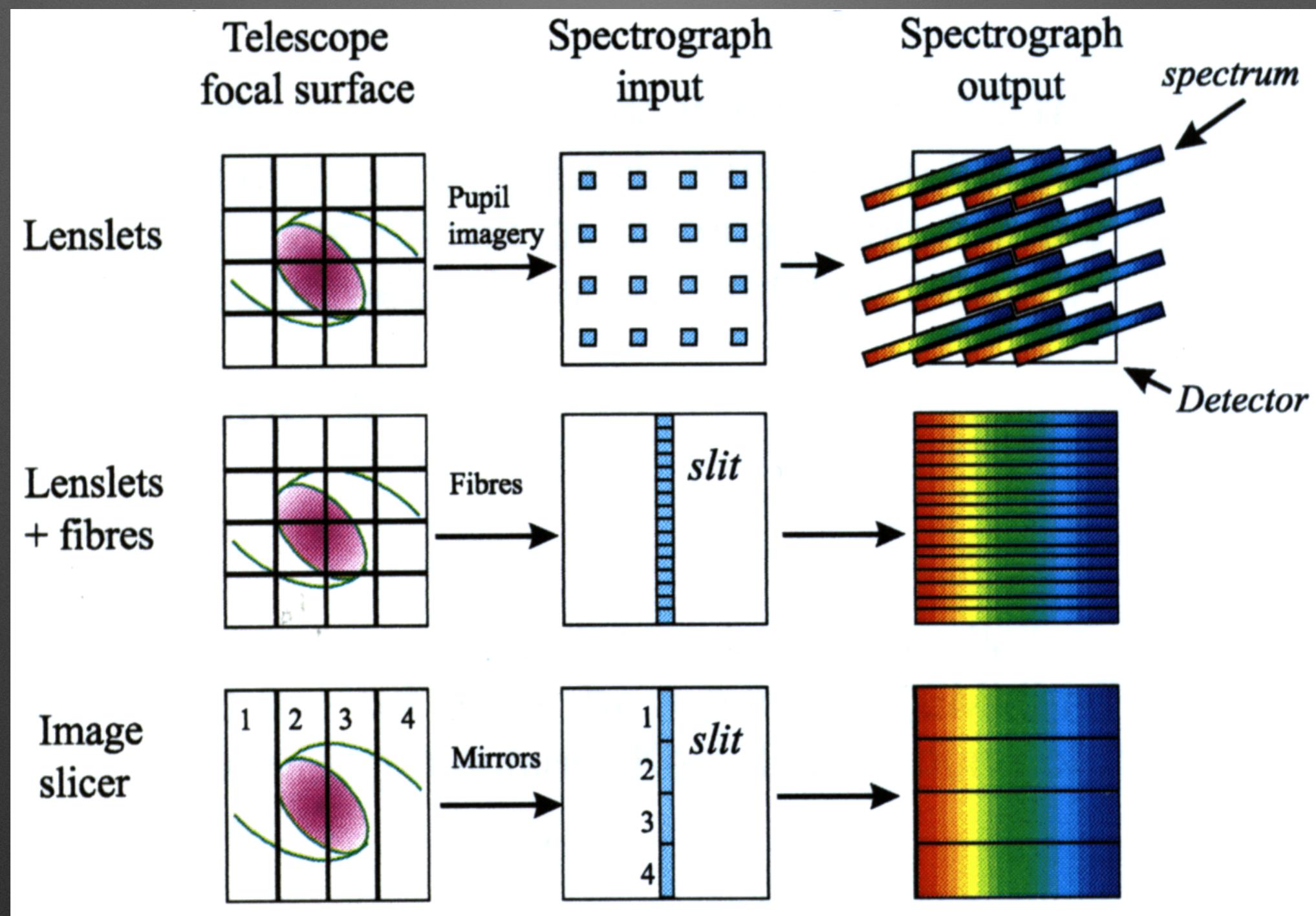
Velocity curve



Velocity field

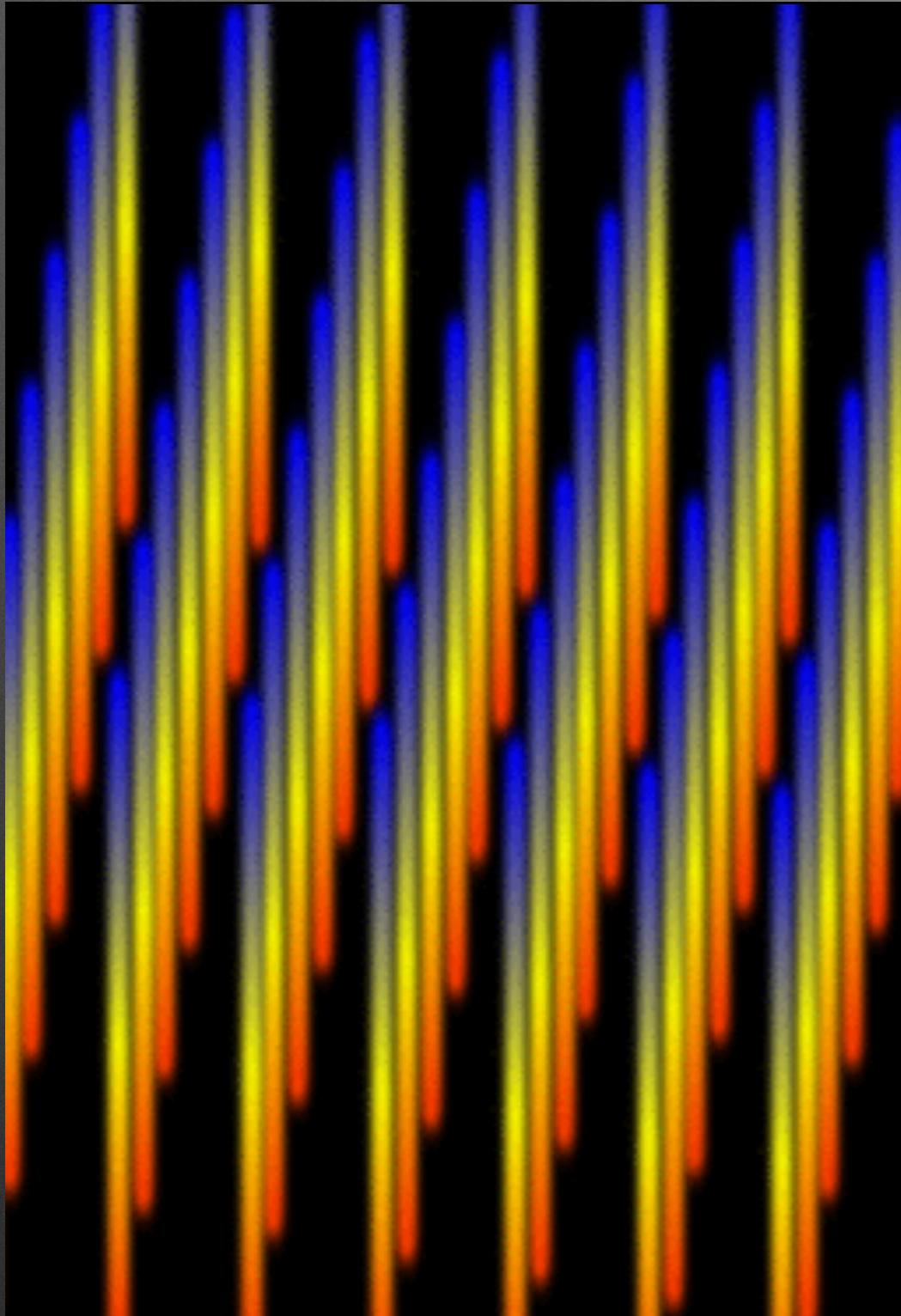


# Types of IFUs



From a talk by Roland Bacon

# Lenslets - SAURON



Uniform illumination at the entrance of the array

The array samples the field and focus the light into micro-pupils

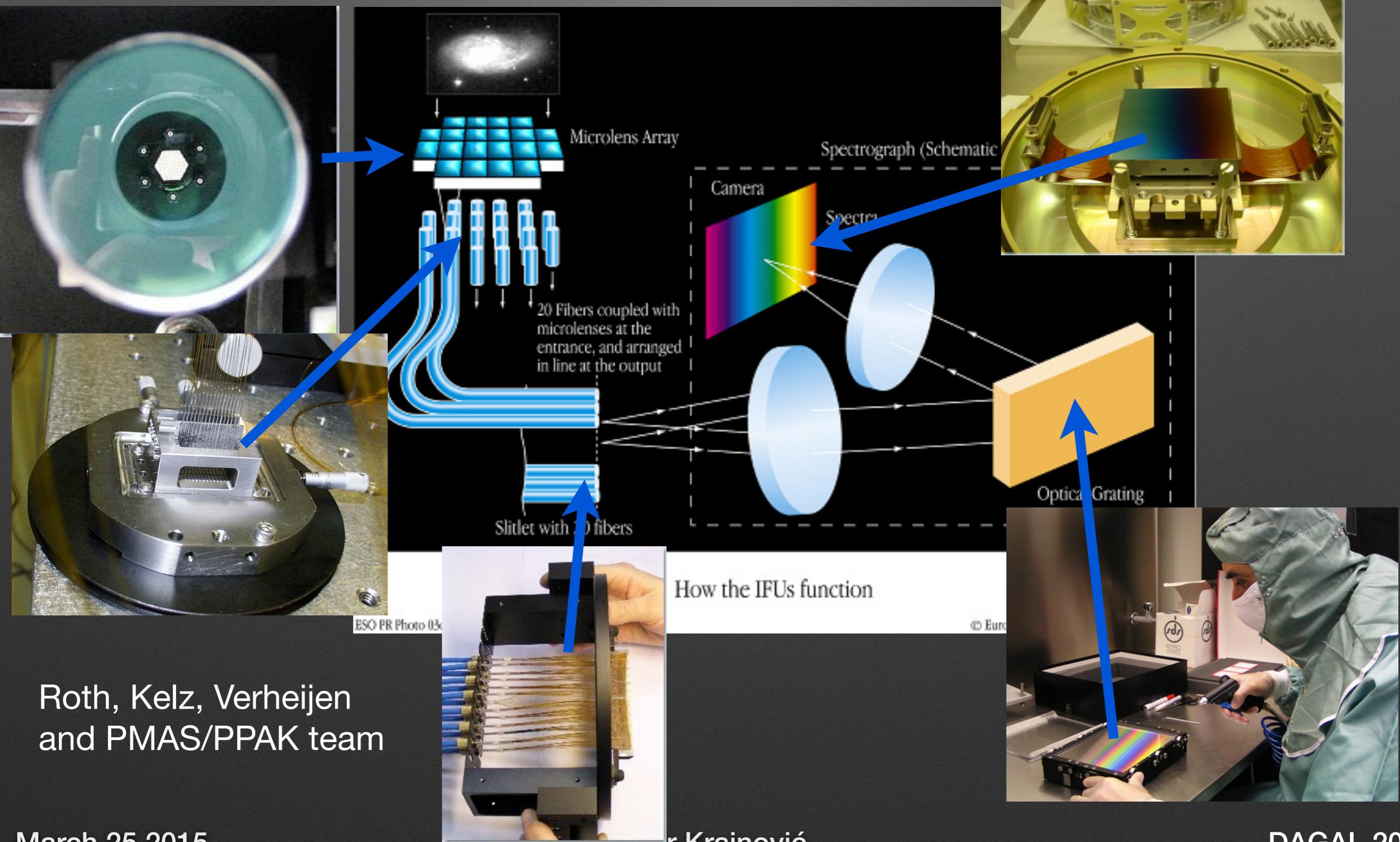
The array is rotated to avoid overlapping between the spectra

The micro-pupils are dispersed via a classical spectrograph

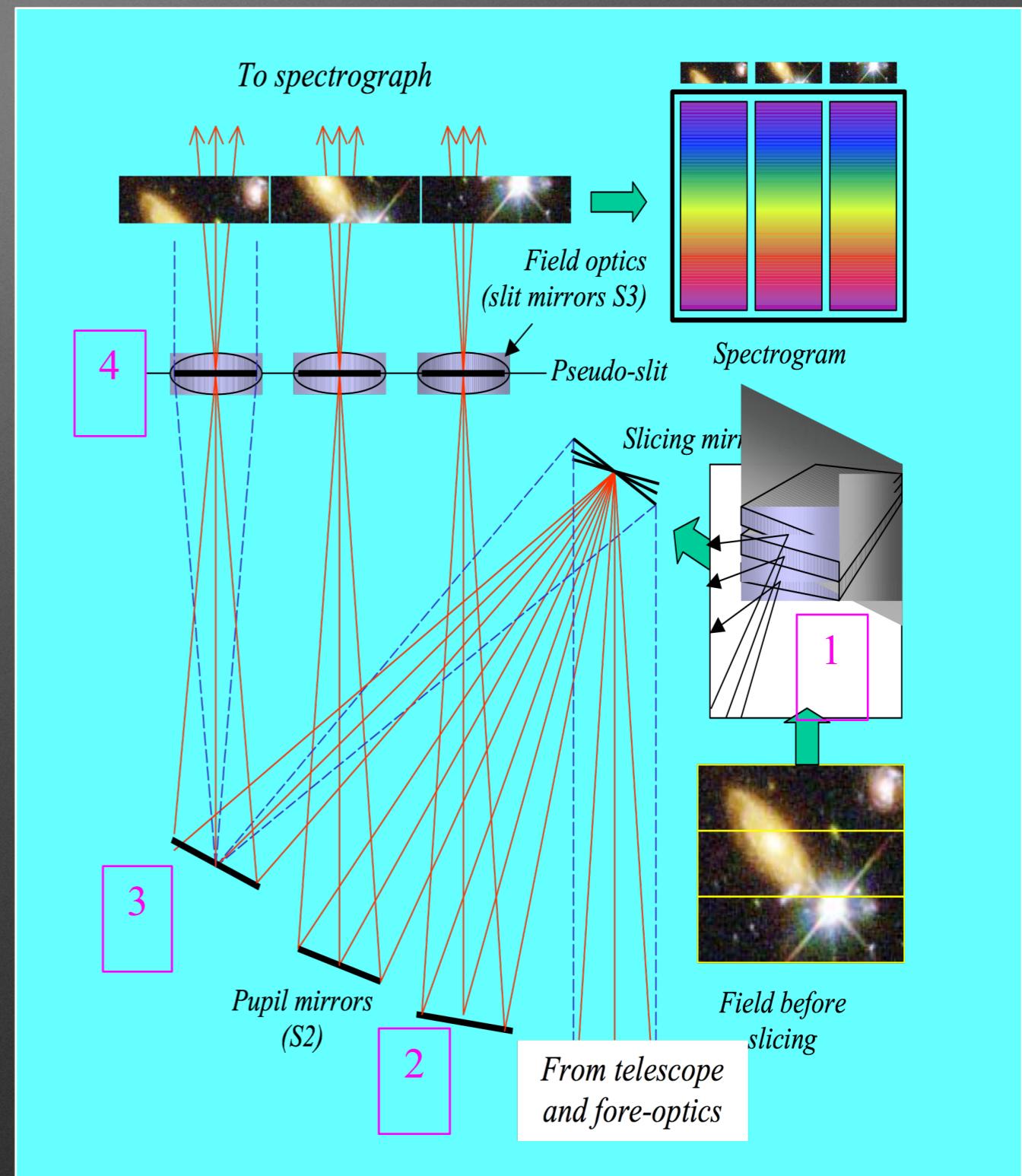
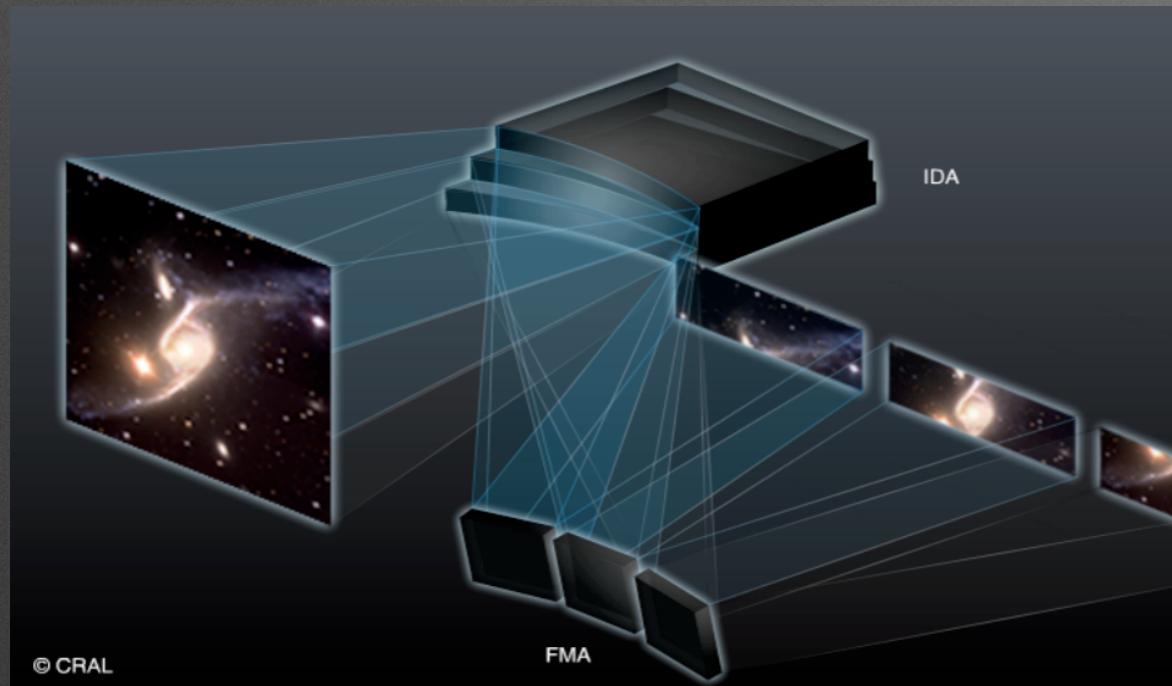
A filter limits the Y range

slide by R. Bacon

# Fibers - PMAS (& PPAK)

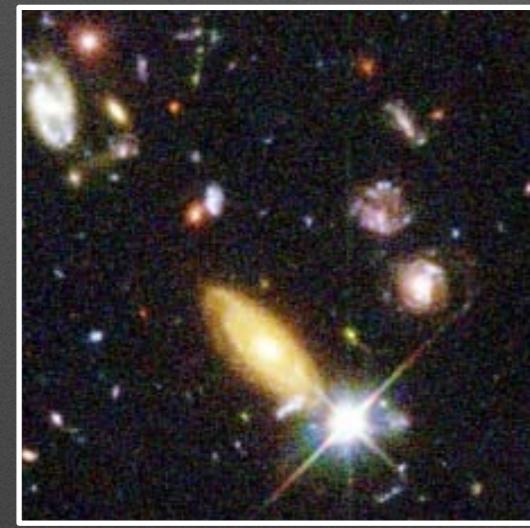


# Slicers - MUSE

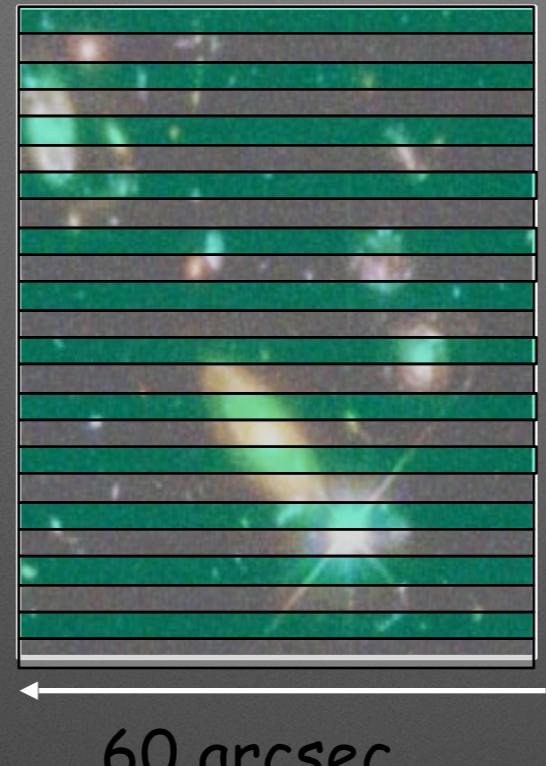


# MUSE - the record holder

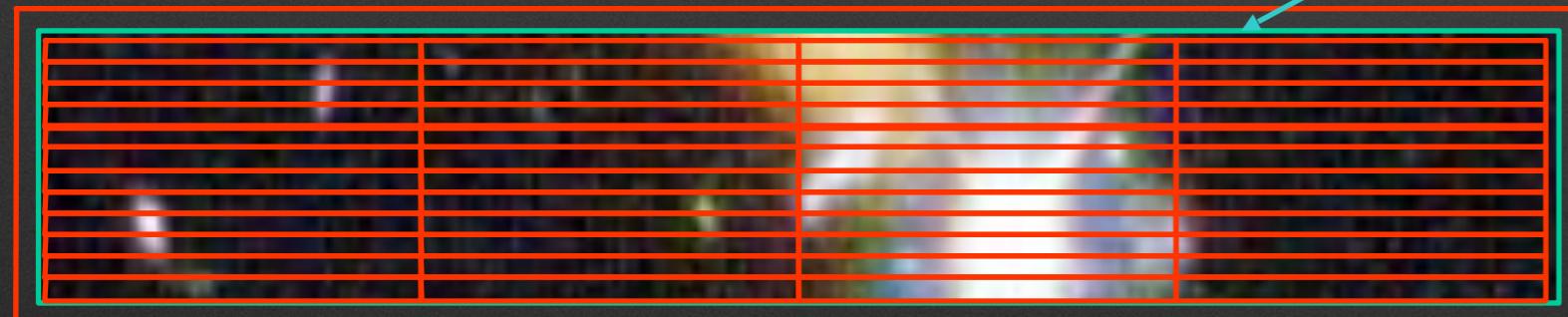
Field-Splitting: in 1x24 separate (IFU) spectrographs



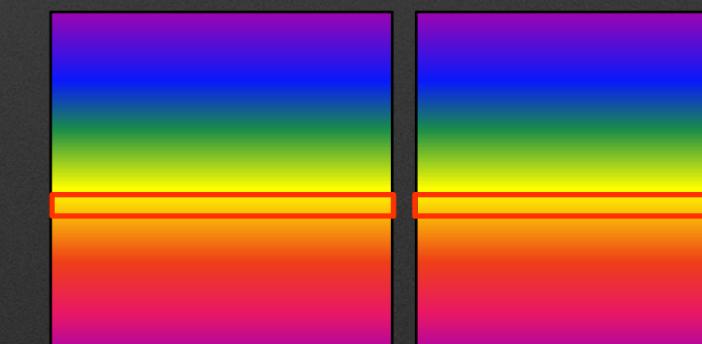
60 arcsec



Slicing: 4 x 12 slices per spectrograph



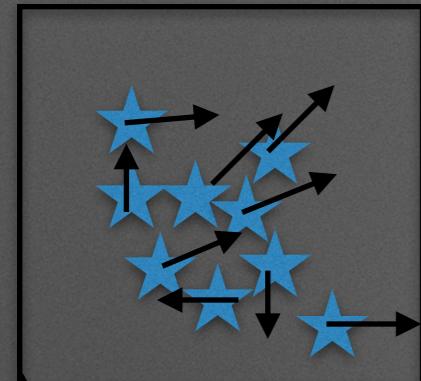
90 000  
spectra per  
observation



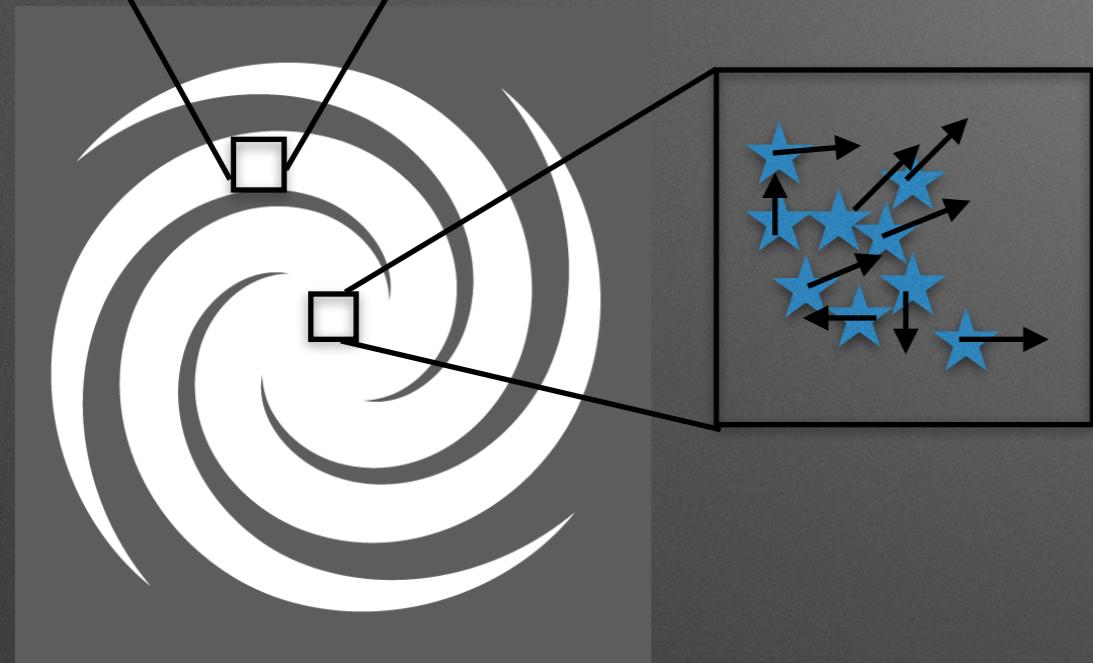
slide by A. Kelz

DAGAL 2015

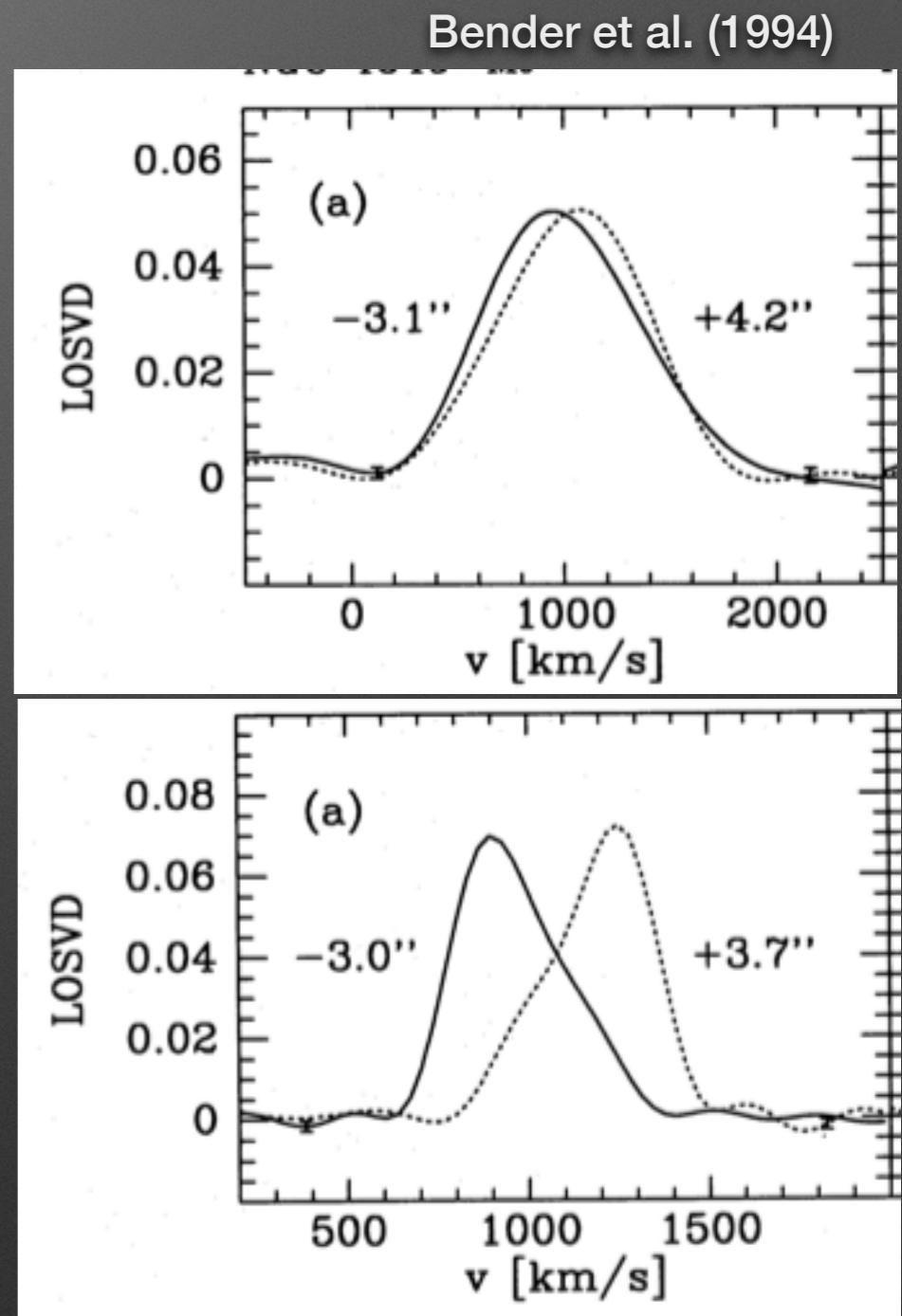
# Methods of extracting kinematics



# What is LOSVD?

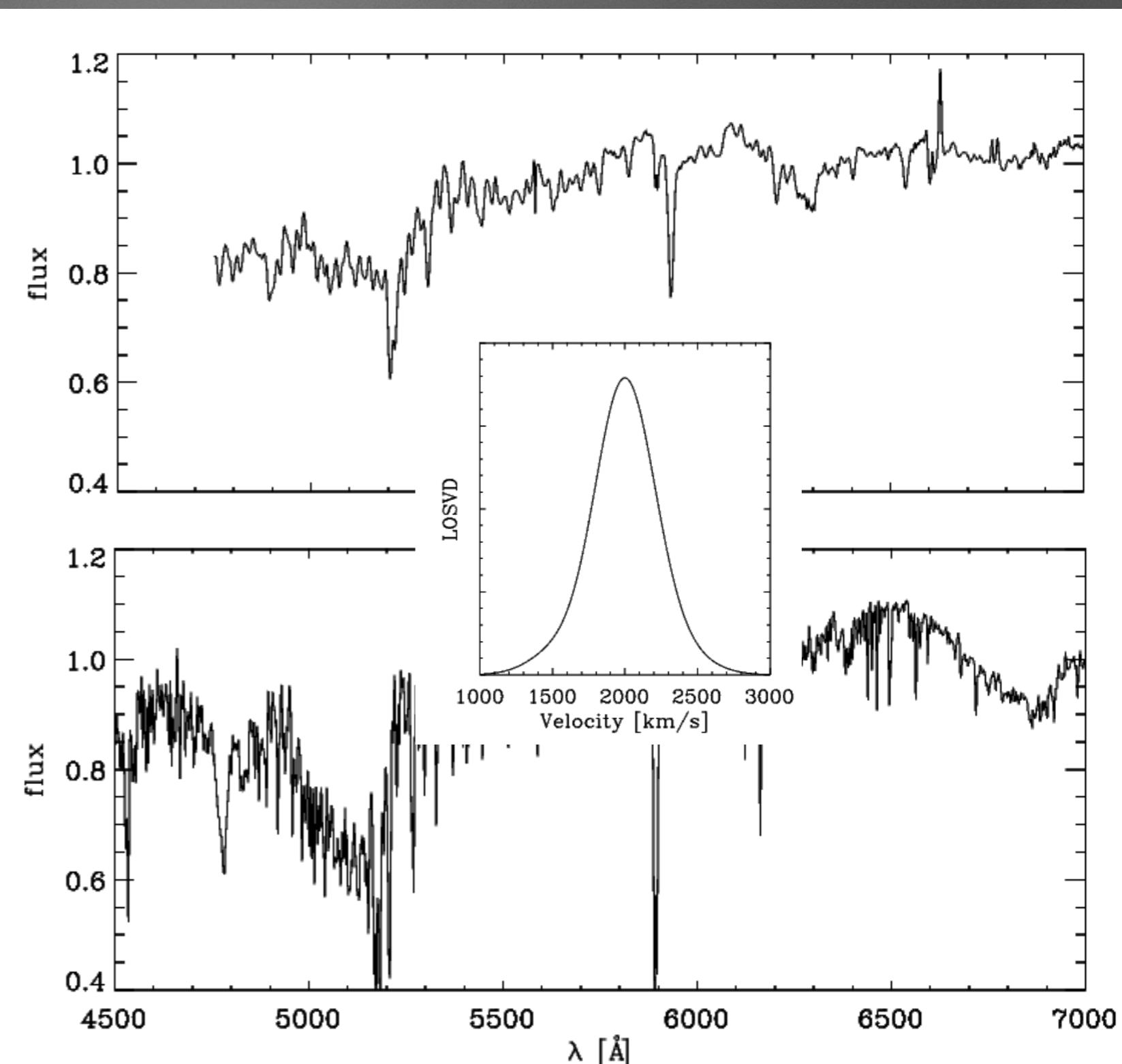


- Line-of-sight velocity distribution (LOSVD)
  - fraction of stars which have velocities between  $v_{\text{LOS}}$  and  $v_{\text{LOS}}+dv_{\text{LOS}}$
  - Gaussian(?) or more complicated in shape
  - not observed directly, but changes the shape of absorption lines



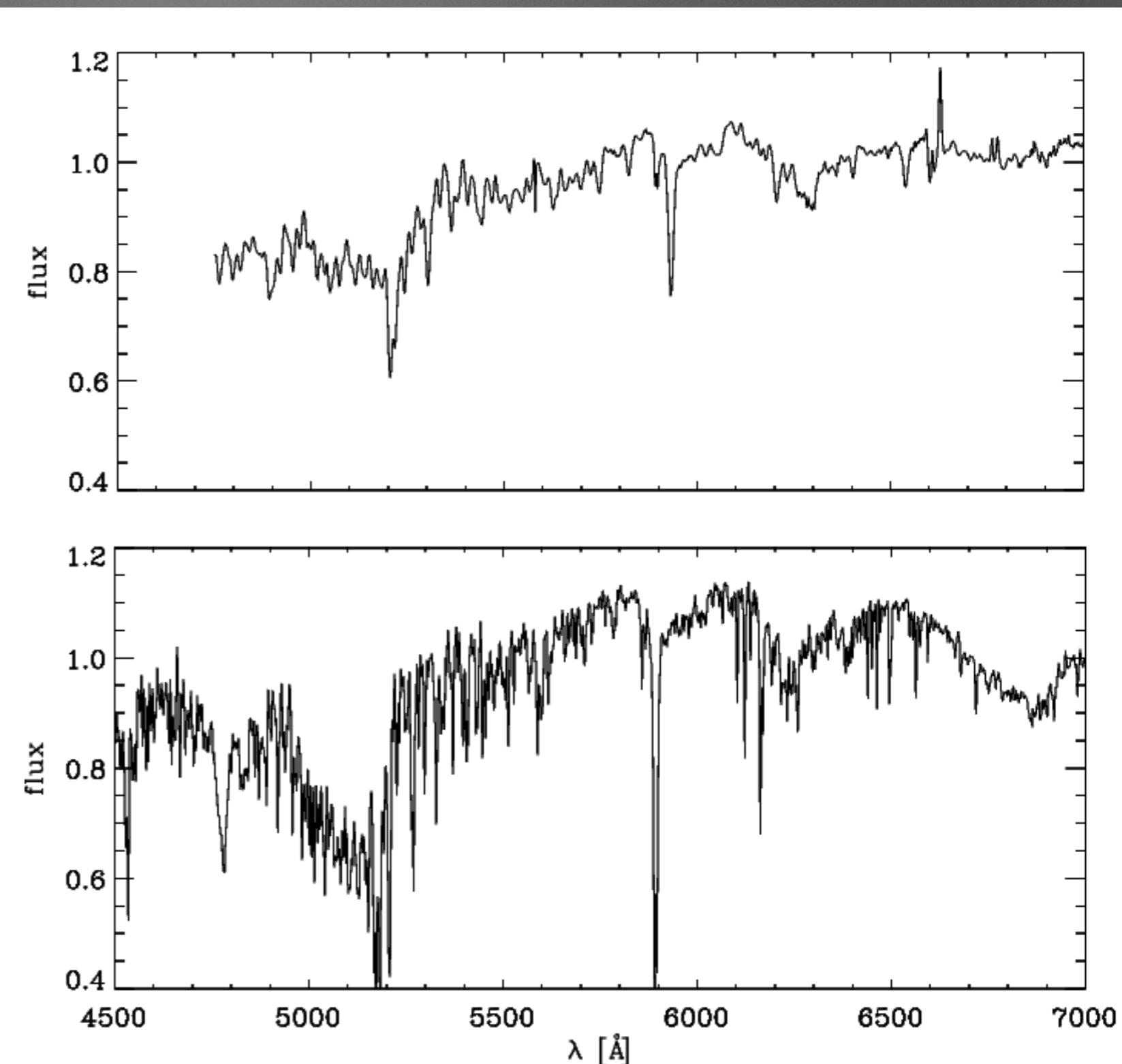
# Recovering LOSVD

- Galaxy spectrum = combination of stellar spectra, Doppler shifted with a certain LOSVD
- 1) find a combination of stars that best reproduce the spectrum
- 2) convolve them with LOSVD and compare with the galaxy spectrum
- 3) minimise the residuals
- 4) parameterise the LOSVD (or use it as is)

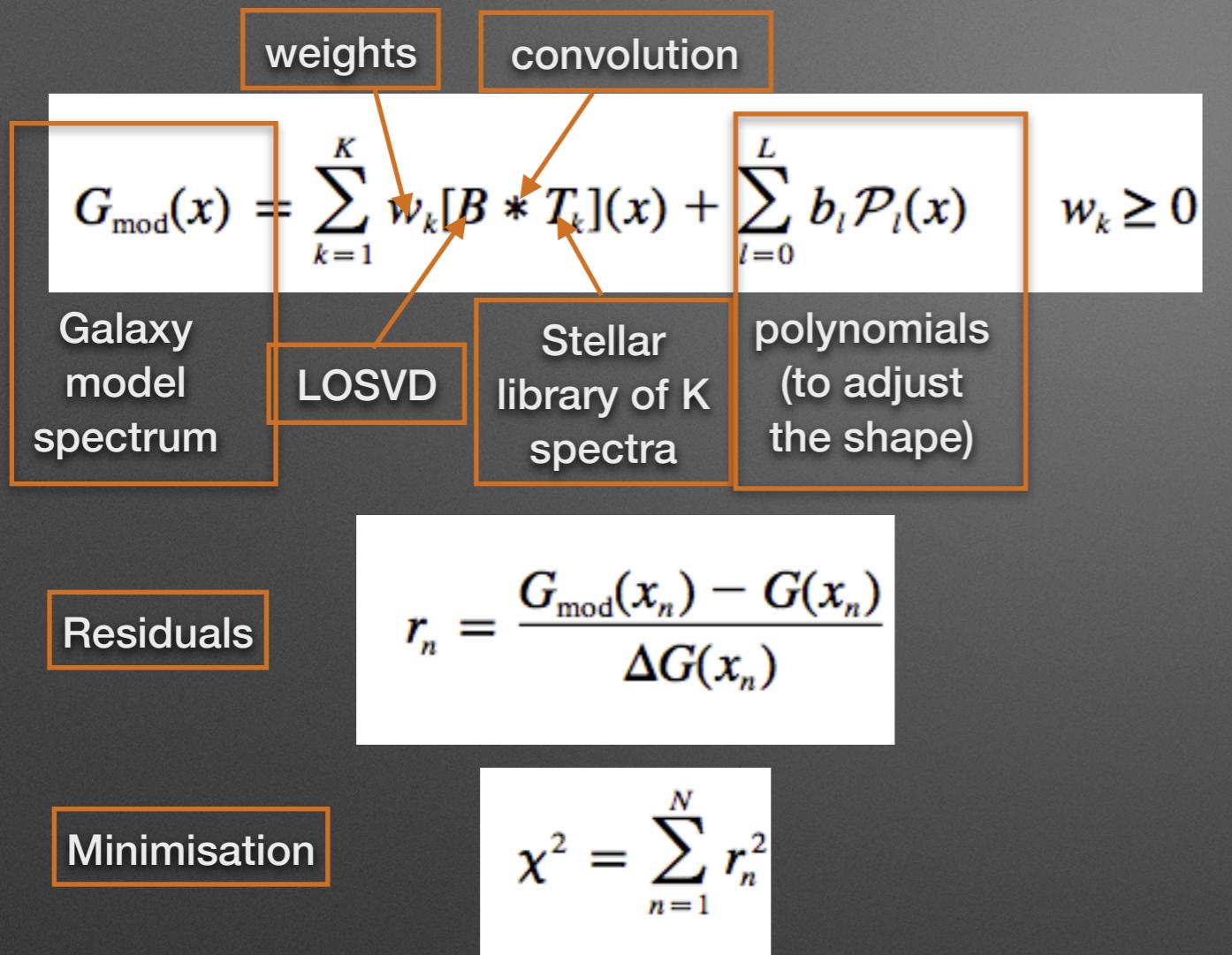


# Recovering LOSVD

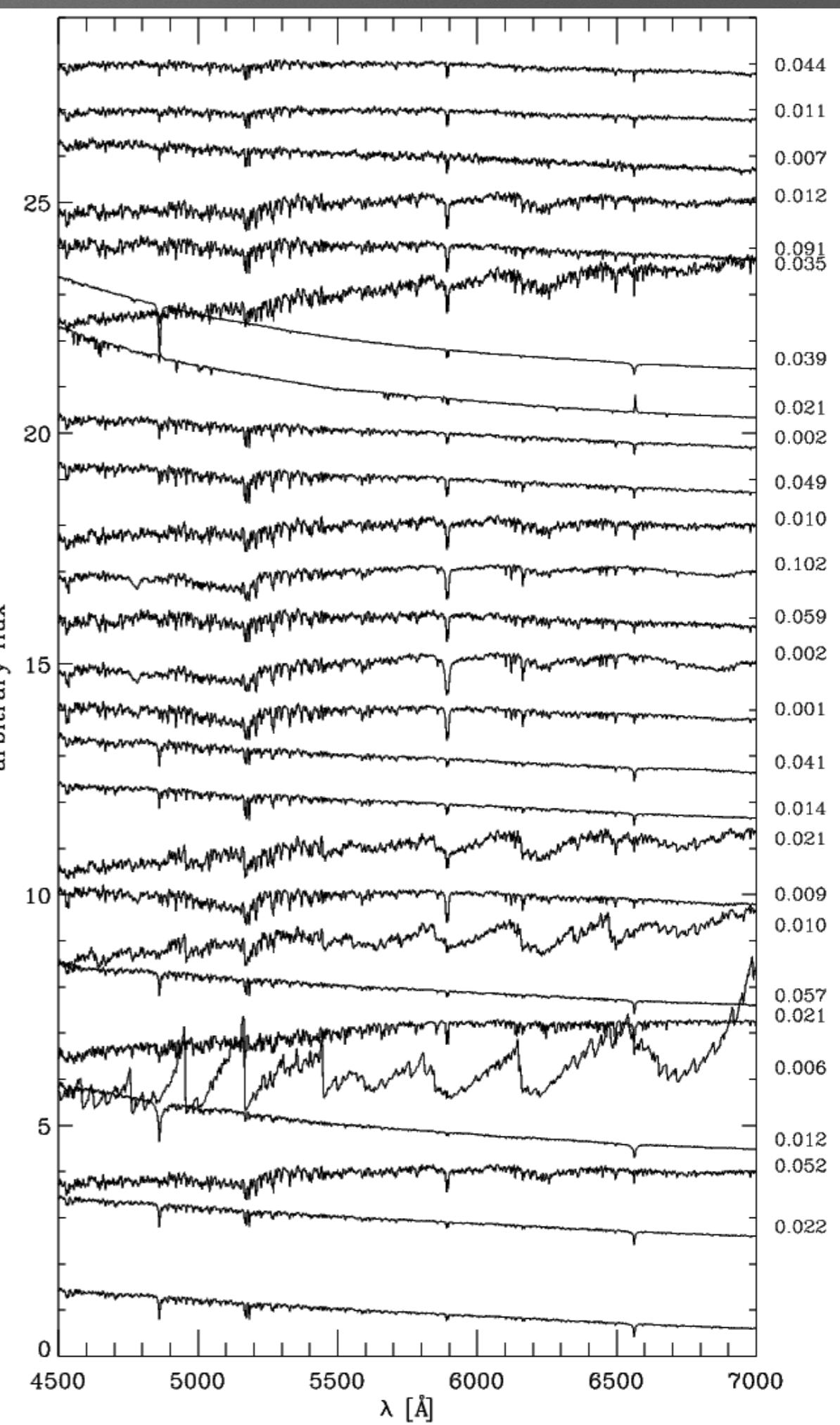
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# Recovering LOSVD

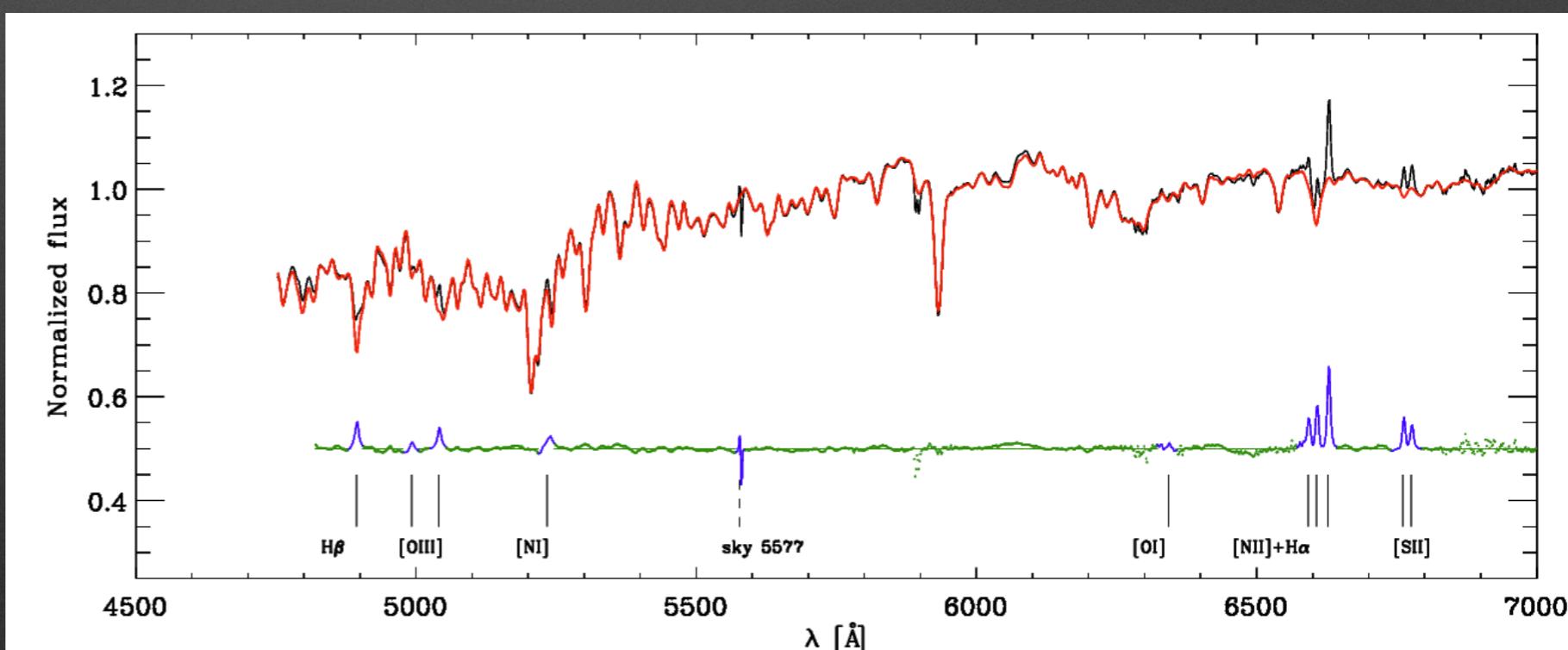


- horrific word: template mismatch
- need: a good library of stellar templates (good resolution, wavelength coverage and covering all sort of stars)



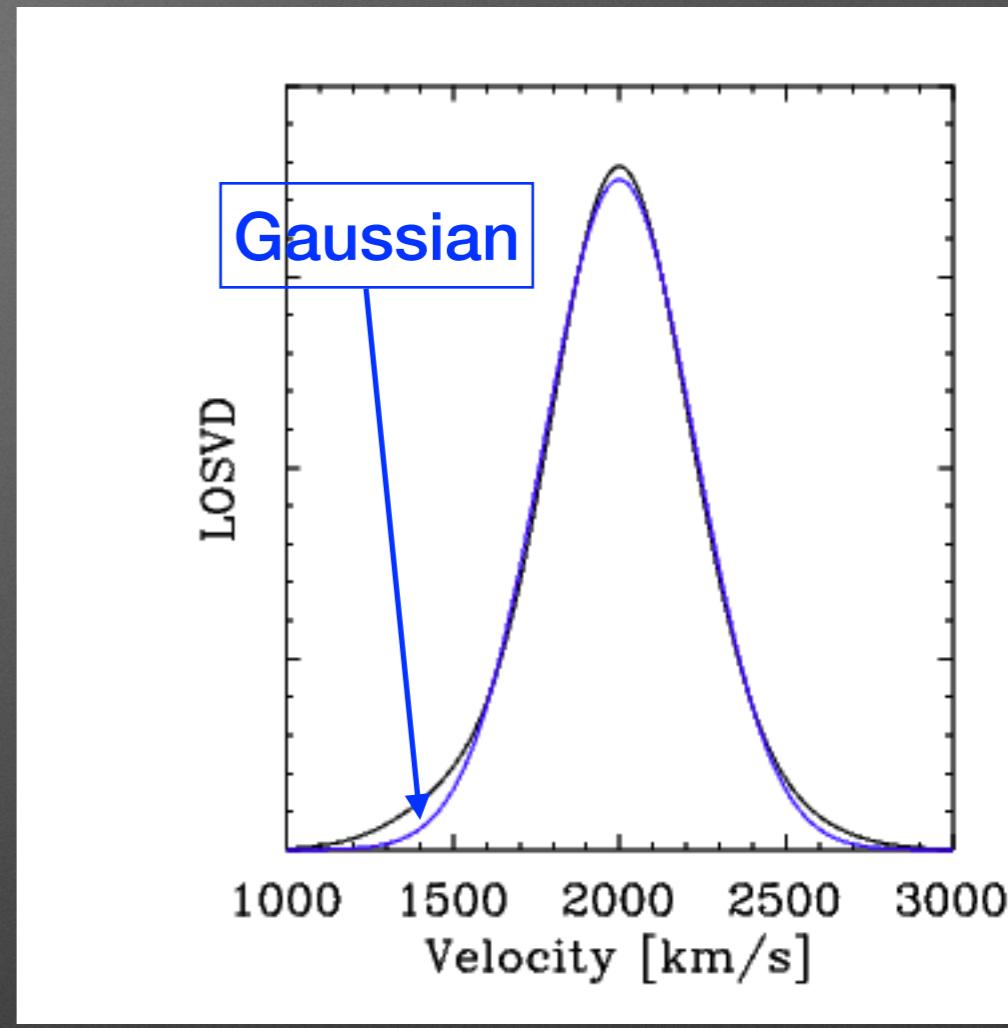
# “Extracting” kinematics

- Different techniques available
  - Fourier correlation quotient (FCQ) (Simkin 1974, Sargent et al. 1977, Franx & Illingworth 1988, Bender et al. 1990)
  - Cross-correlation (XC or CCF) (Tonry & Davis 1979, Statler et al. 1995)
  - maximum penalized likelihood (MPL) (Saha & Williams 1994, Merritt 1997, Pinkney et al. 2003)
  - direct fitting in pixel space (Rix&White 1992, Kuijen&Merrifield 1993, van der Marel 1994; Saha & Williams 1994; Merritt 1997; Gebhardt et al. 2000; Kelson et al. 2000, Cappellari & Emsellem 2004)
- merits of direct fitting:
  - easy masking of emission-lines or bad pixels (e.g. sky-lines)
  - easier to account for template mismatch
  - convolving stellar spectra to fit galaxies



# Parameterising the LOSVD

- simple assumption: LOSVD is Gaussian
  - no theoretical reason for this
  - good 1st order approximation, but not sufficiently accurate
  - accurate LOSVDs are required for dynamical models
- multiple Gaussians (e.g. Franx & Illingworth 1988, Rix & White 1992, Kuijken & Merrifield 1993)
- Gauss-Hermite series (van der Marel & Franx 1993, Gerhard (1993))



$$L_{\text{GH}}(v) = \frac{\gamma}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{1}{2} \left( \frac{v - V}{\sigma} \right)^2 \right] \sum_{i=0}^N h_i H_i \left( \frac{v - V}{\sigma} \right)$$

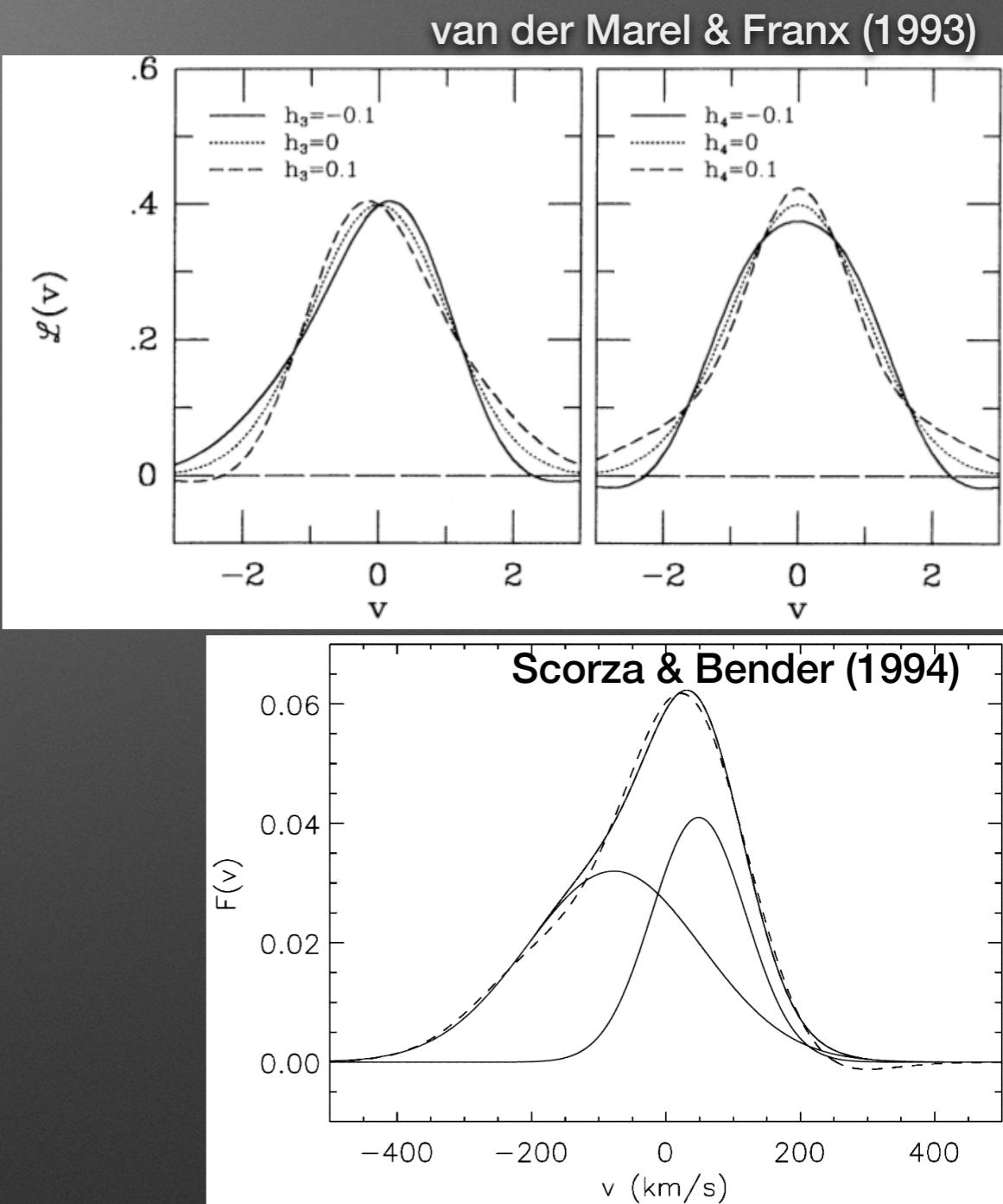
from Cappellari et al. (2002)

$$H_0(y) = 1, \quad H_1(y) = \sqrt{2}y, \quad H_2(y) = \frac{2y^2 - 1}{\sqrt{2}}, \quad H_3(y) = \frac{y(2y^2 - 3)}{\sqrt{3}},$$

$$H_4(y) = \frac{y^2(4y^2 - 12) + 3}{2\sqrt{6}}, \quad H_5(y) = \frac{y[y^2(4y^2 - 20) + 15]}{2\sqrt{15}}, \quad H_6(y) = \frac{y^2[y^2(8y^2 - 60) + 90] - 15}{12\sqrt{5}}$$

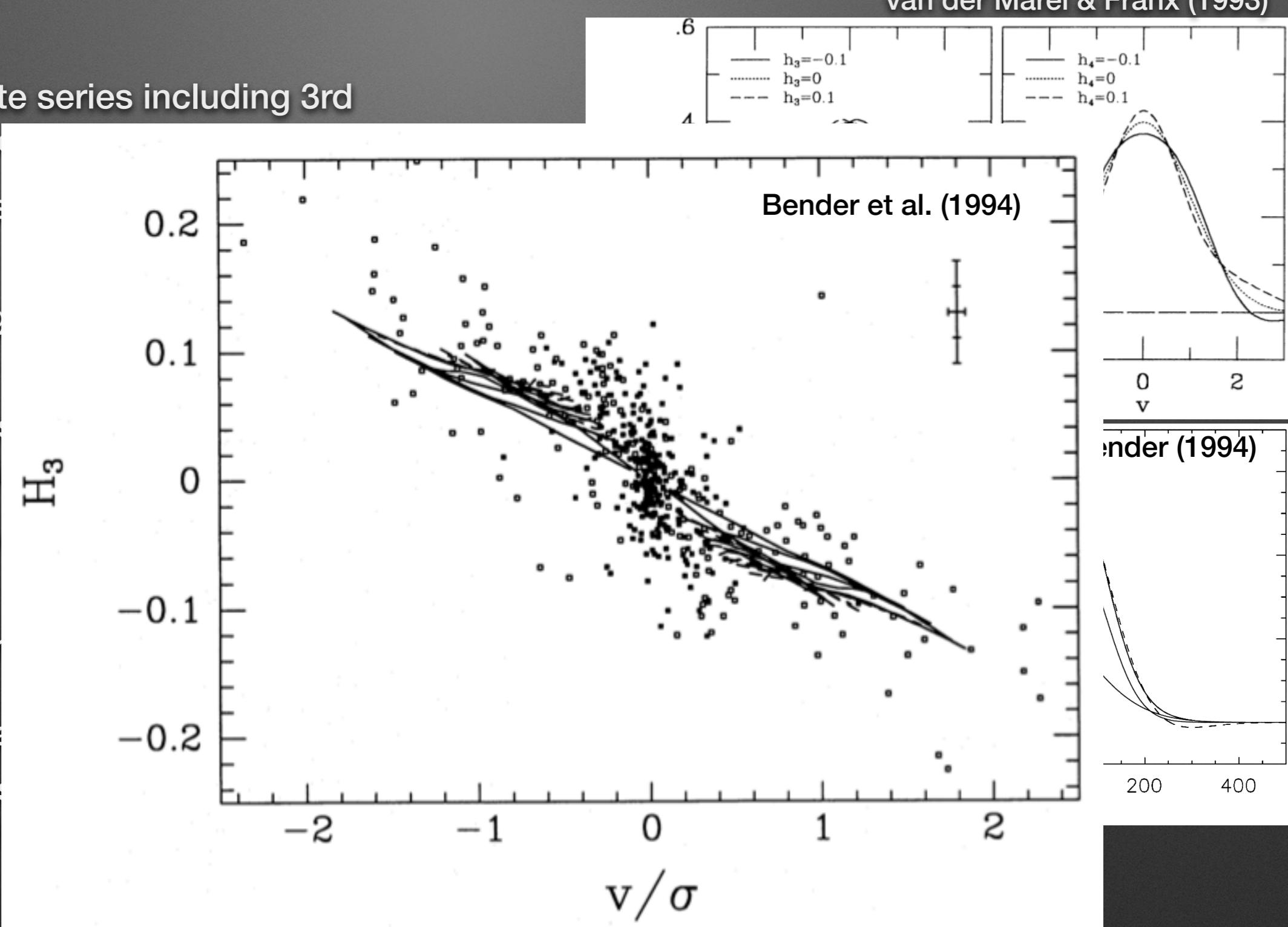
# Understanding the LOSVD

- Gauss-Hermite series including 3rd and 4th moment standardly used
- $h_3$  - skewness : departure from symmetry
  - strong in galaxies with discs (e.g. Bender et al. 1994)
- $h_4$  - kurtosis: symmetric departures from a Gaussian
- works best if departures are not very strong!
- non-parametric LOSVDs also used (Merritt 1997, Pinkney et al. 2003, Houghton et al. 2006...)
  - histograms (directly fitted by the dynamical models)



# Understanding the LOSVD

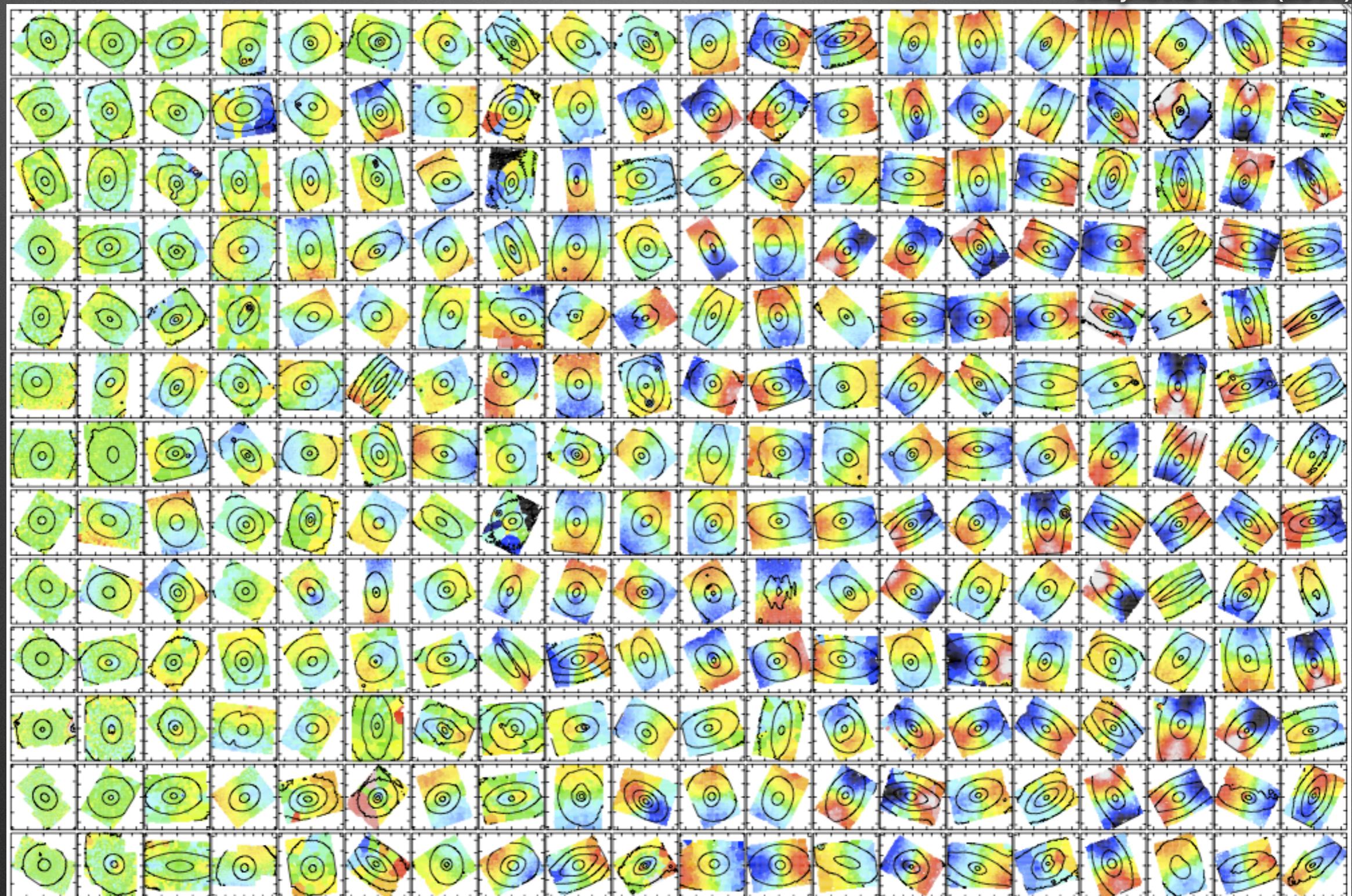
- Gauss-Hermite series including 3rd and 4th moment
- $h_3$  - skewness symmetry
  - strong in galaxies - Bender et al.
- $h_4$  - kurtosis: symmetric about zero from a Gaussian
- works best if asymmetries are strong!
- non-parametric fit (Merritt 1997, Houghton et al.)
  - histograms of velocity distributions



# Stellar kinematics of early-type galaxies

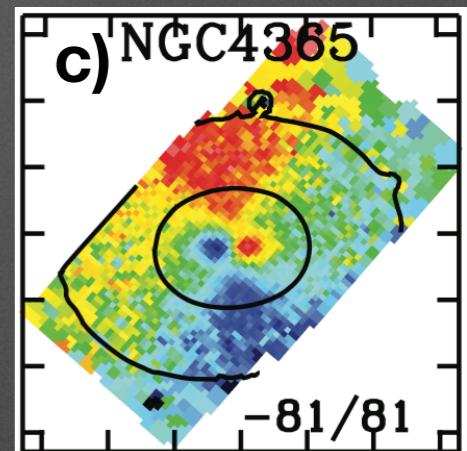
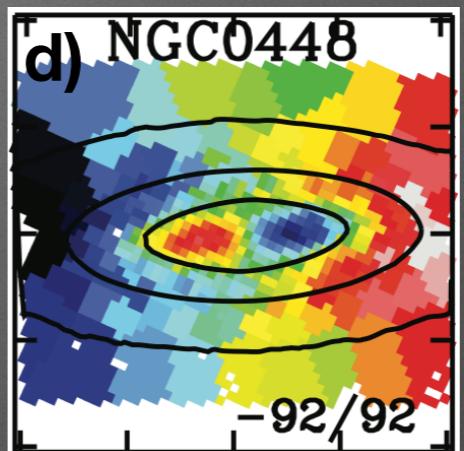
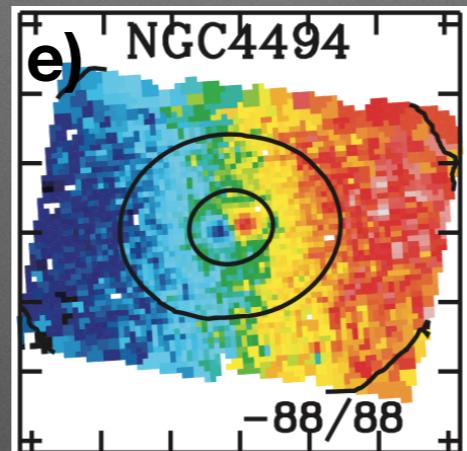
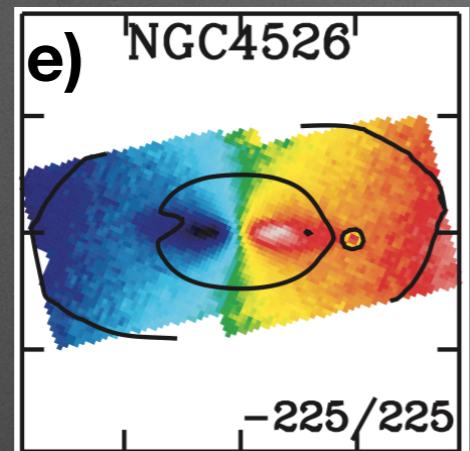
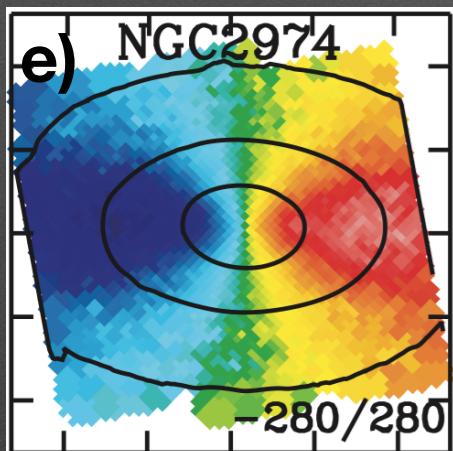
# Velocity (fields) maps

Krajnović et al. (2011)

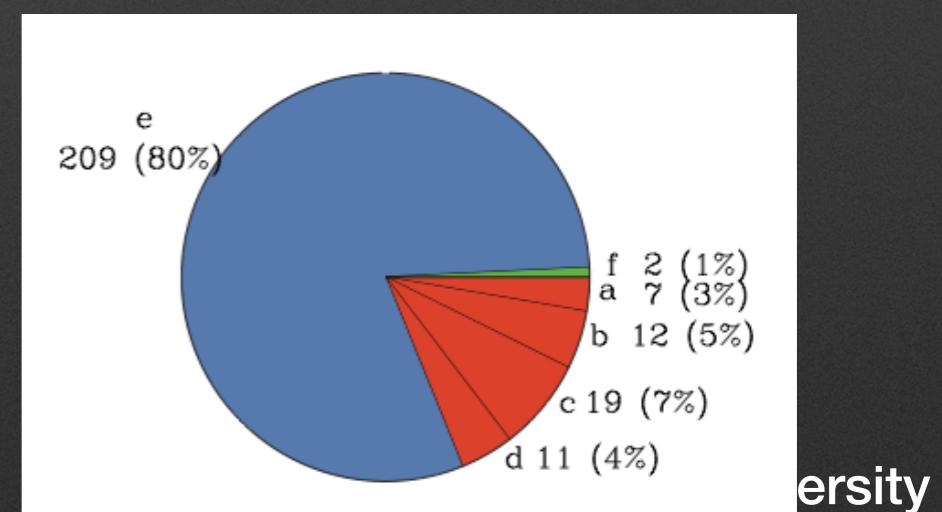
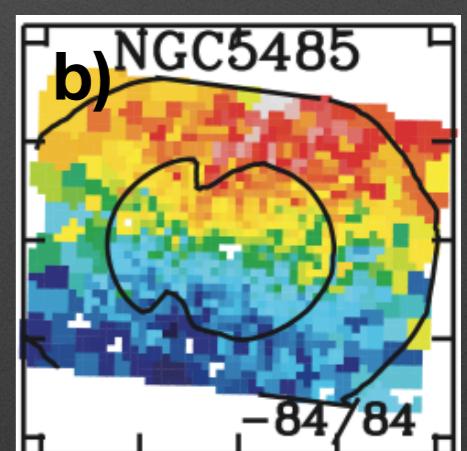
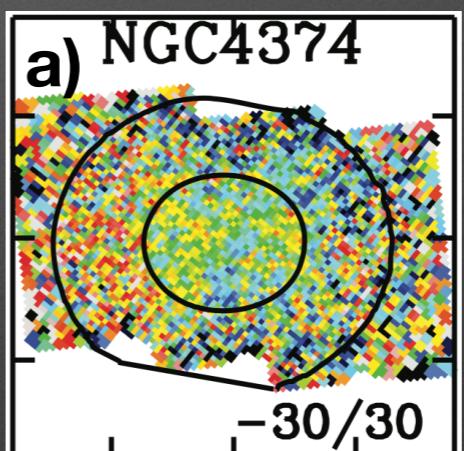


# Kinematic features

maps from ATLAS<sup>3D</sup> survey (Krajnović et al. 2011)



- velocity maps of early-type galaxies show ample evidence for complex kinematic structures
- mostly regular (simple & “boring”)
- what is their origin?
- what do they tell us about the mass assembly?

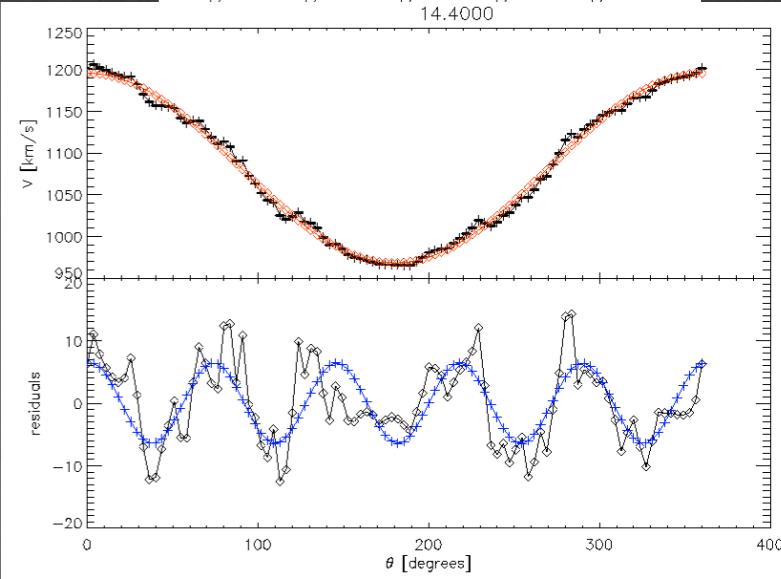
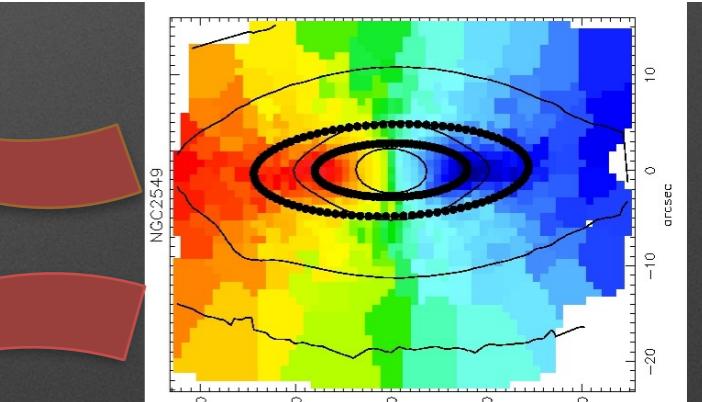
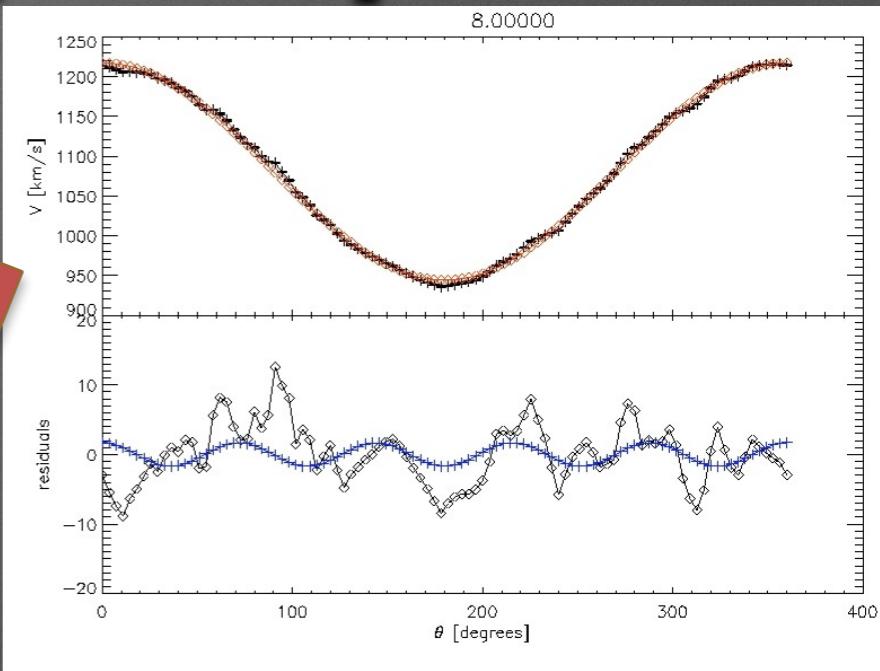


# Analysis of velocity maps

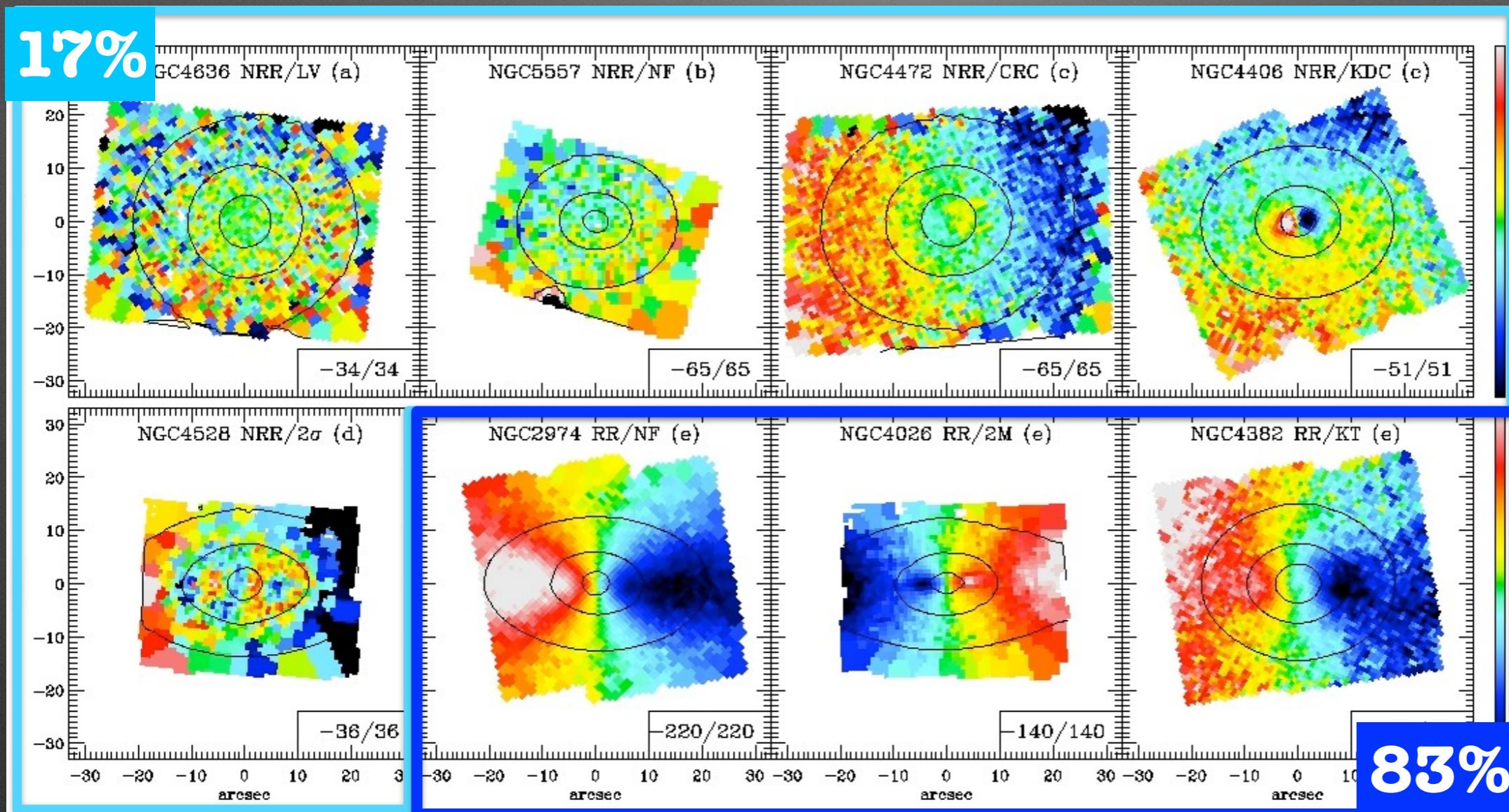
- (Thin) discs have regular velocity maps
- Along an ellipse (inclined circle) velocity is

$$V = V_0 + V_R \cos(\theta)$$

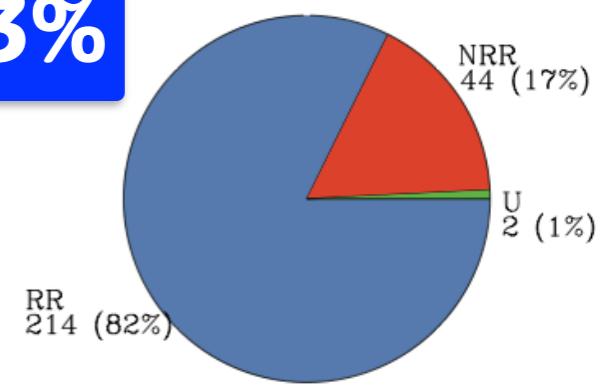
- Kinometry (Krajnović et al. 2006): generalising the surface photometry (e.g. Jedrezejewski 1987) to higher-order moments of the LOSVD
- ~80% of ETGs have disk-like velocity maps at 4%



# Types of velocity maps

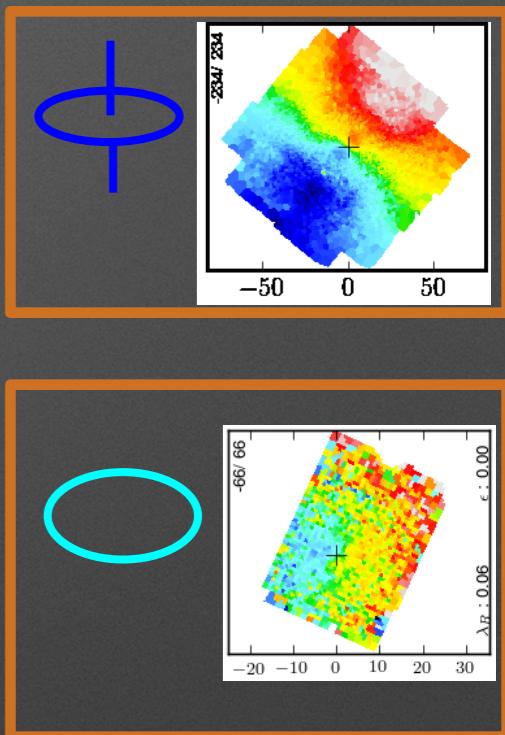
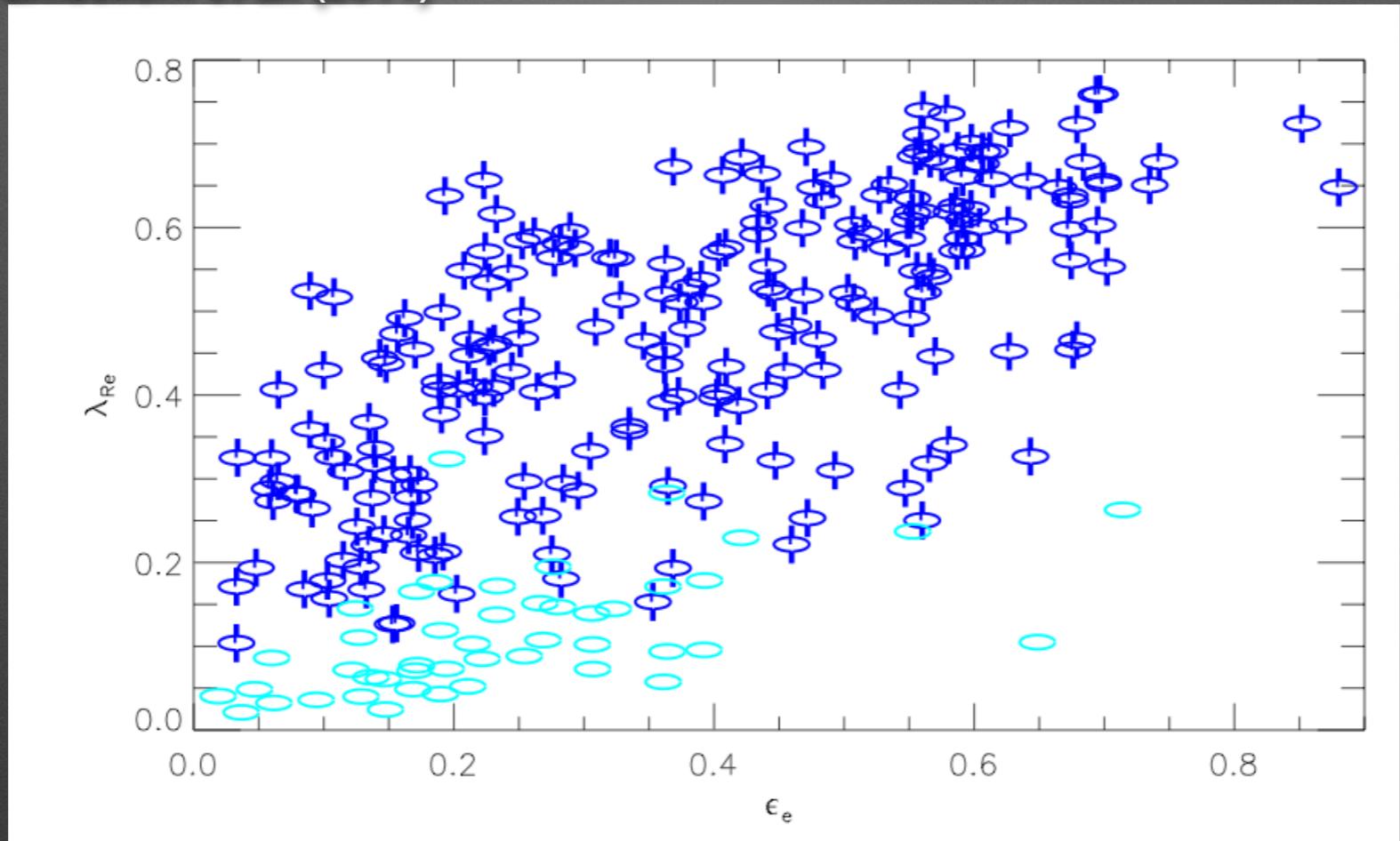


- What (else) does kinematics tell us about galaxies?



# Specific angular momentum - $\lambda_R$

Emsellem et al. (2011)

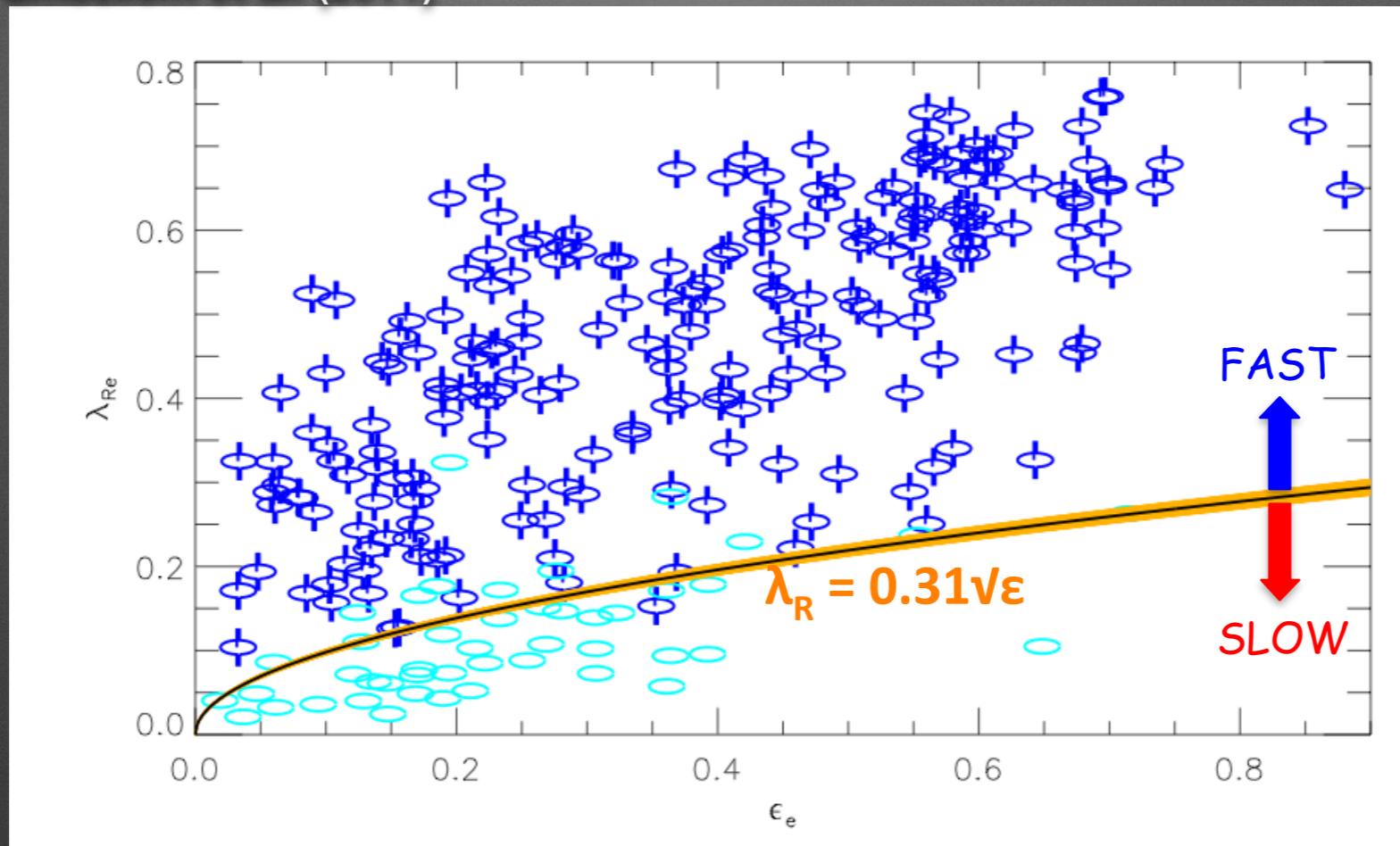


- Kinematic classification using **specific angular momentum**,  $\lambda_R$  (Emsellem et al. 2007)
- Fast rotators: Regular, disk-like velocity maps
- Slow rotators: Non-regular velocity maps, KDCs, no net rotation

$$\lambda_R = \frac{\langle R \cdot |V| \rangle}{\langle R \sqrt{V^2 + \sigma^2} \rangle}$$

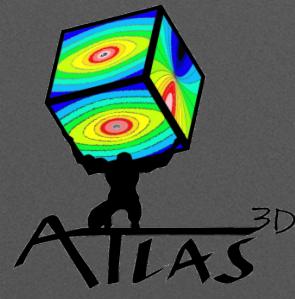
# Specific angular momentum - $\lambda_R$

Emsellem et al. (2011)



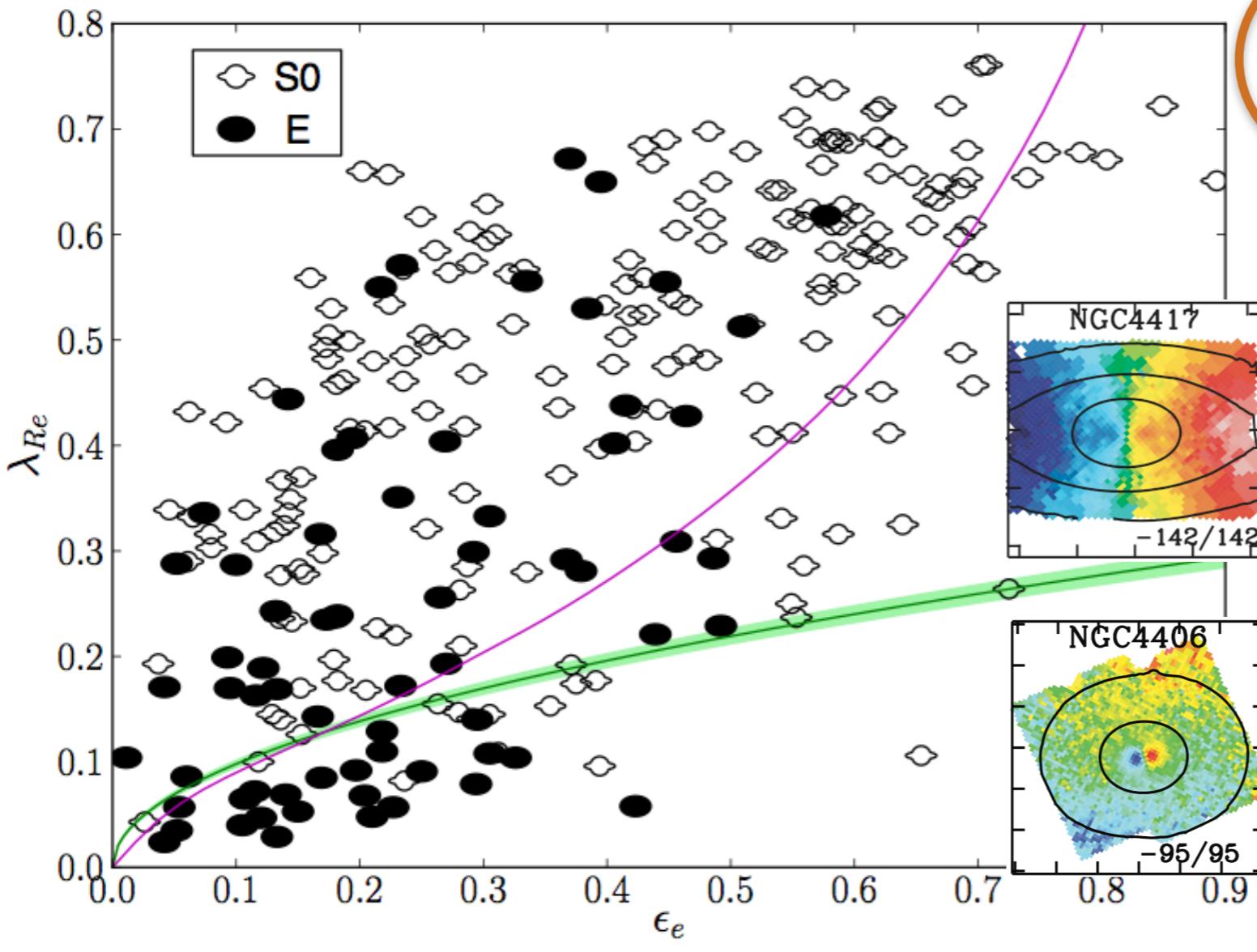
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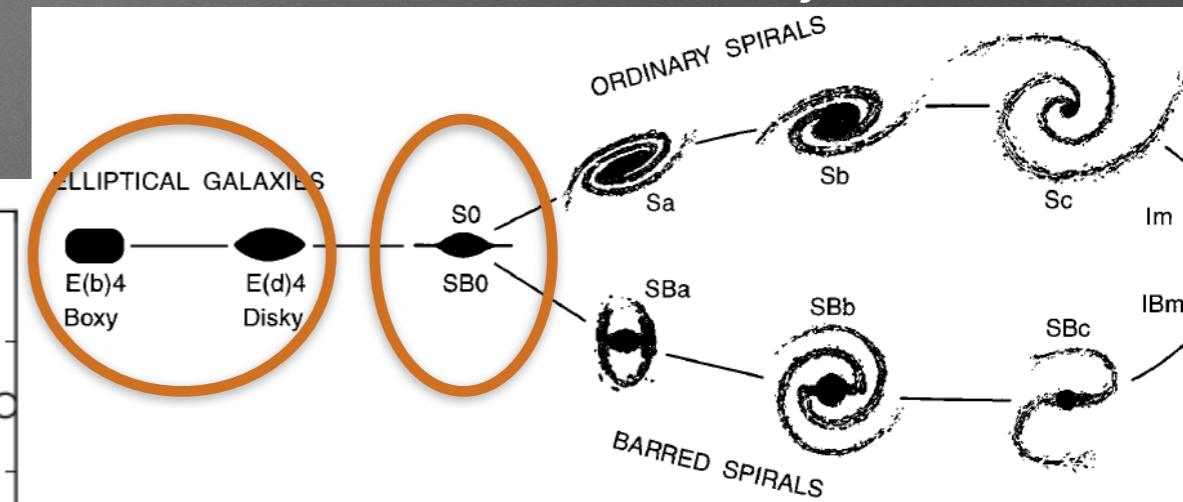


# $\lambda_R$ vs Hubble classes

Emsellem et al. (2011)



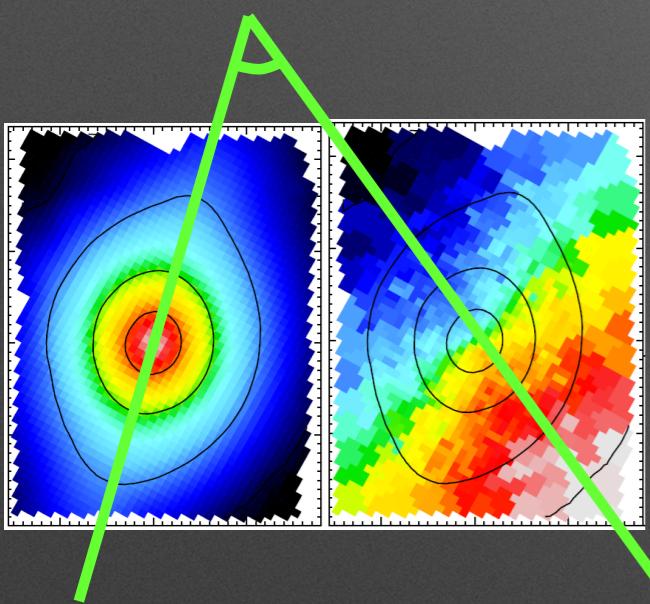
Kormendy & Bender 1996



- 66% of E are FR
- 20% of FR are E
- $FR \approx S0 + E(d)$
- SR = true ellipticals

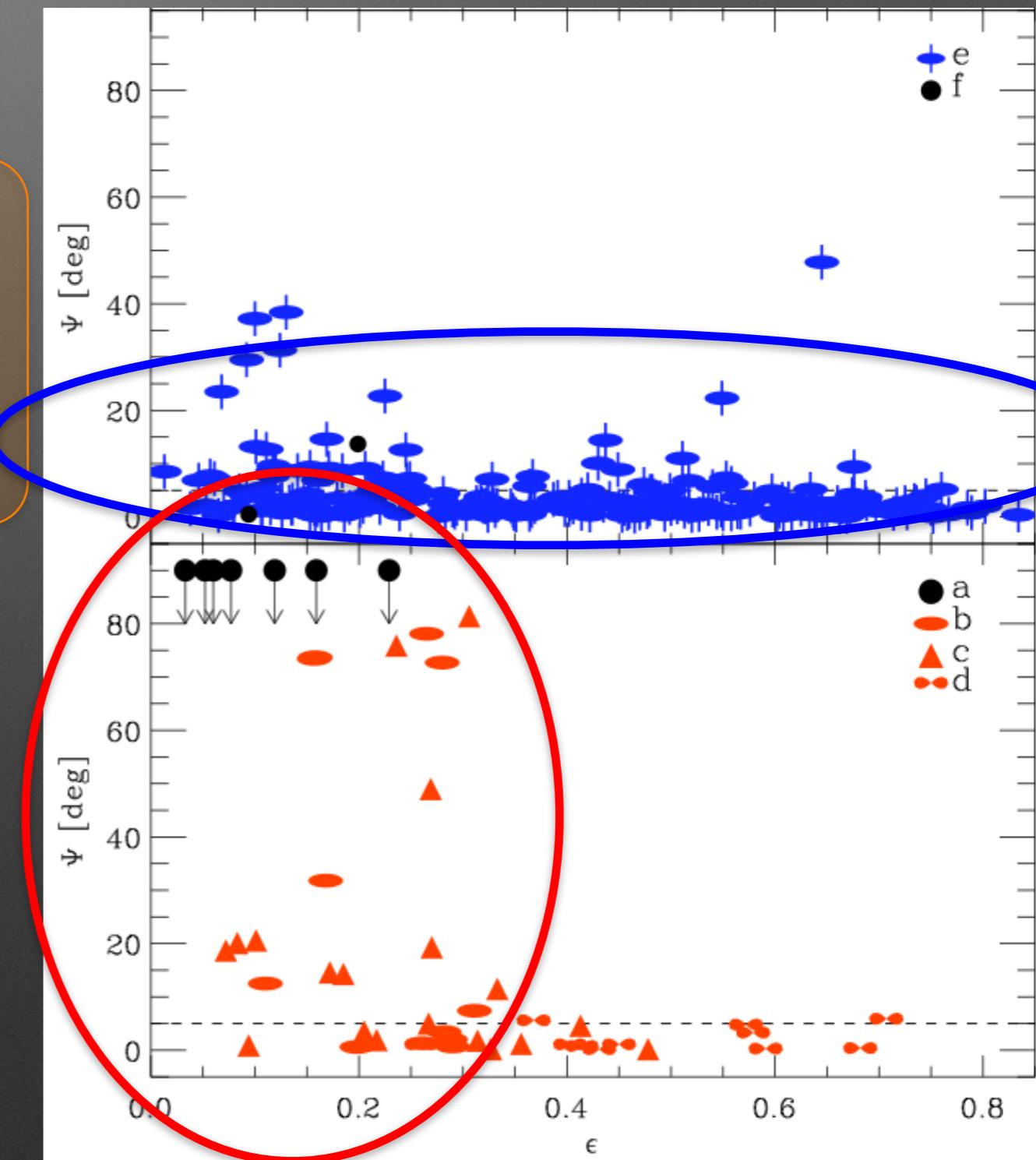
# Kinematic misalignment

Krajnović et al. (2011)



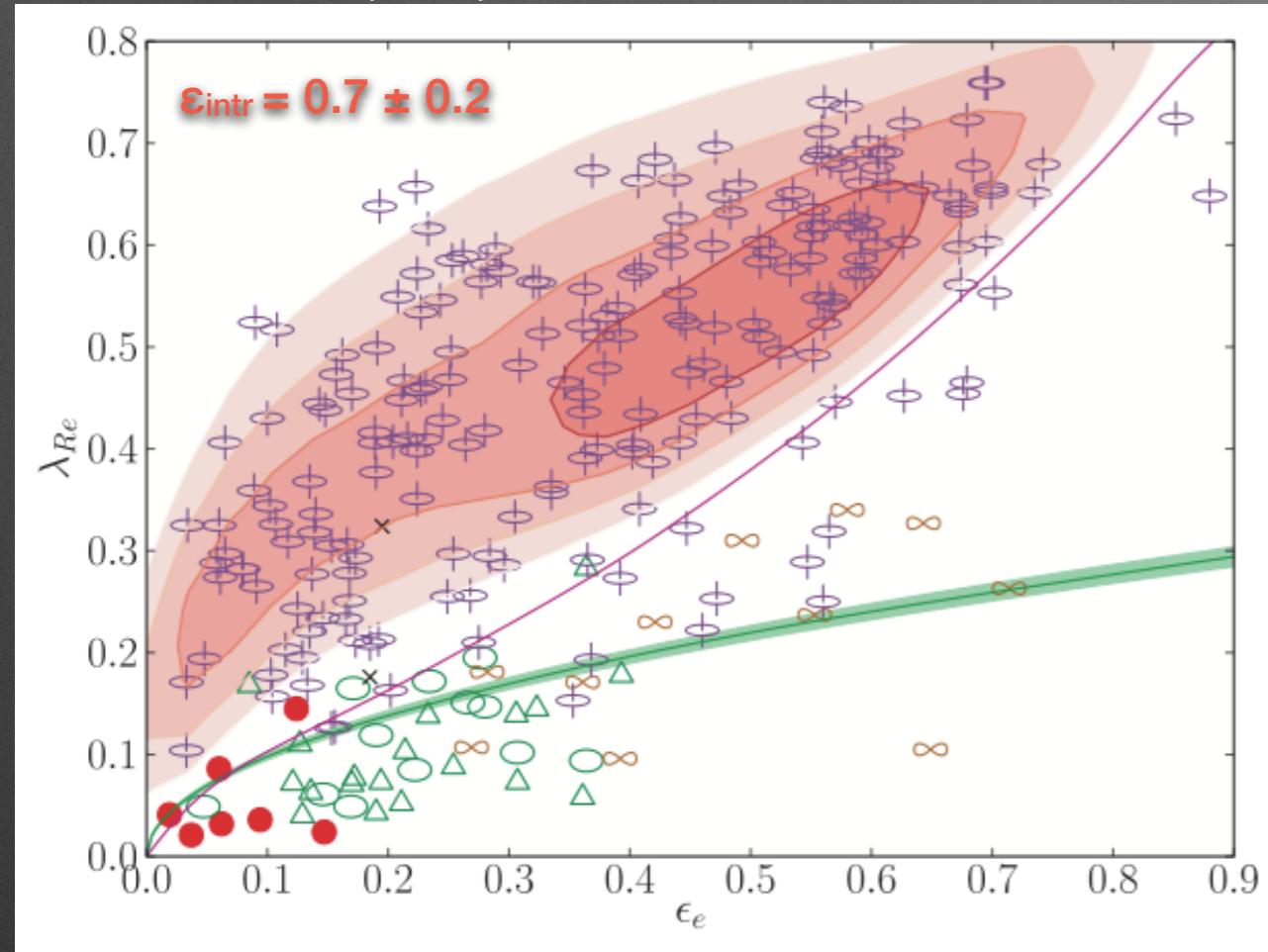
**Aligned:**  
 $\Psi < 5^\circ$  71%  
 $\Psi < 10^\circ$  84%  
 $\Psi < 15^\circ$  90%

- FR: aligned  $\rightarrow$  nearly axisymmetric systems (+ bars!)
- SR: (also) misaligned  $\rightarrow$  triaxial systems

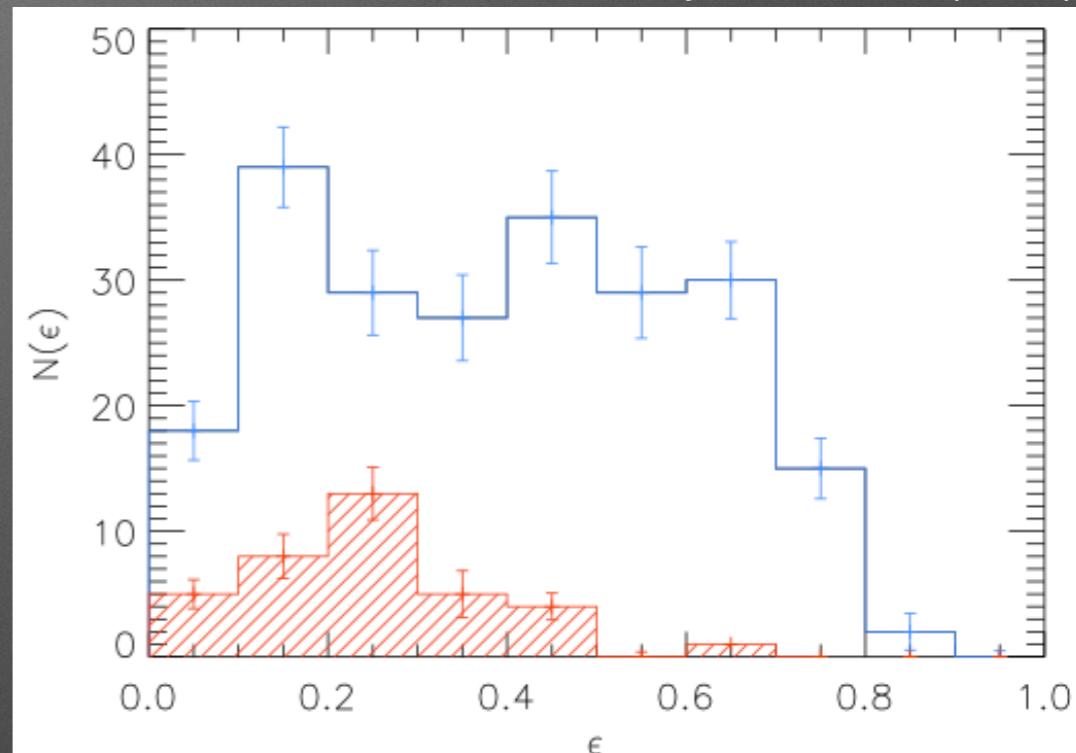


# Shape from kinematics

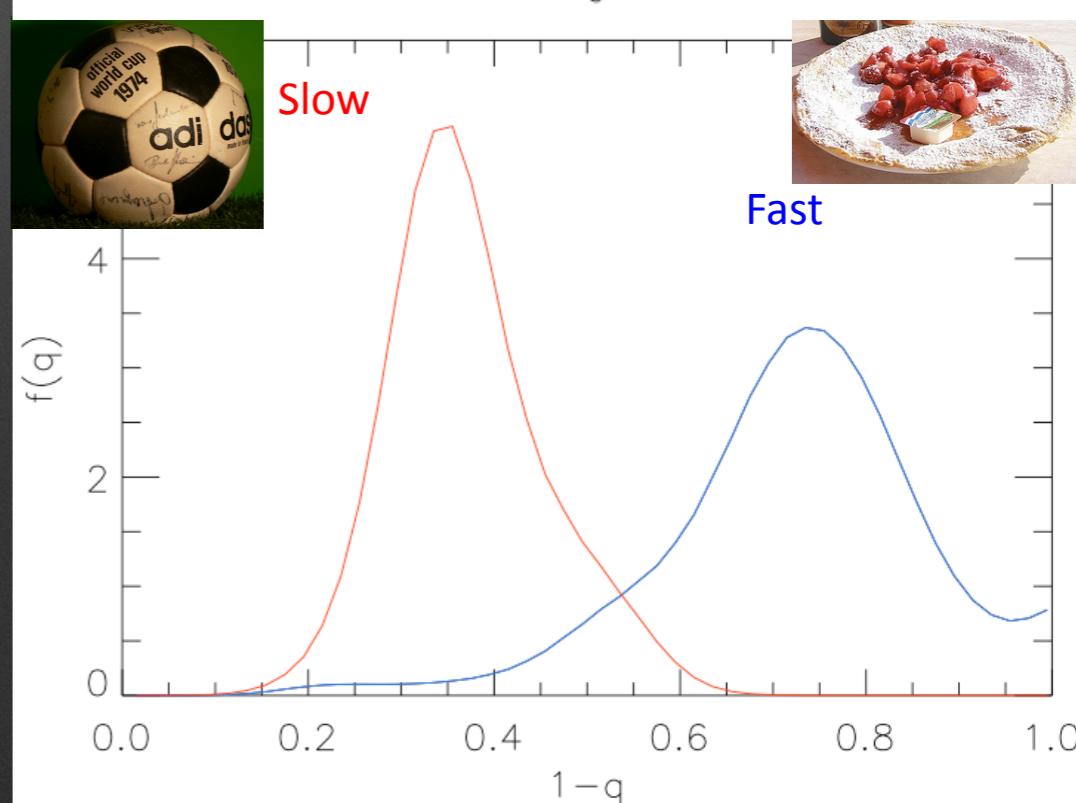
Emsellem et al. (2014)



Weijmans et al. (2014)

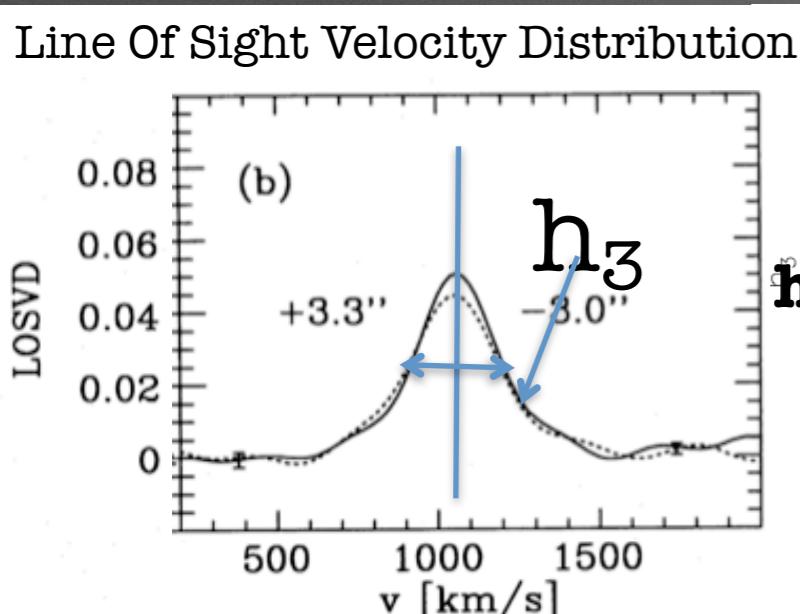


- FR shape:  $q = 0.25 \pm 0.14$  - similar to spirals (Lambas+1992, Padilla & Strauss 2008)
- SR shape:  $q = 0.63 \pm 0.09$  - rounder than previous studies of Es (e.g. Tremblay & Merritt 1996)

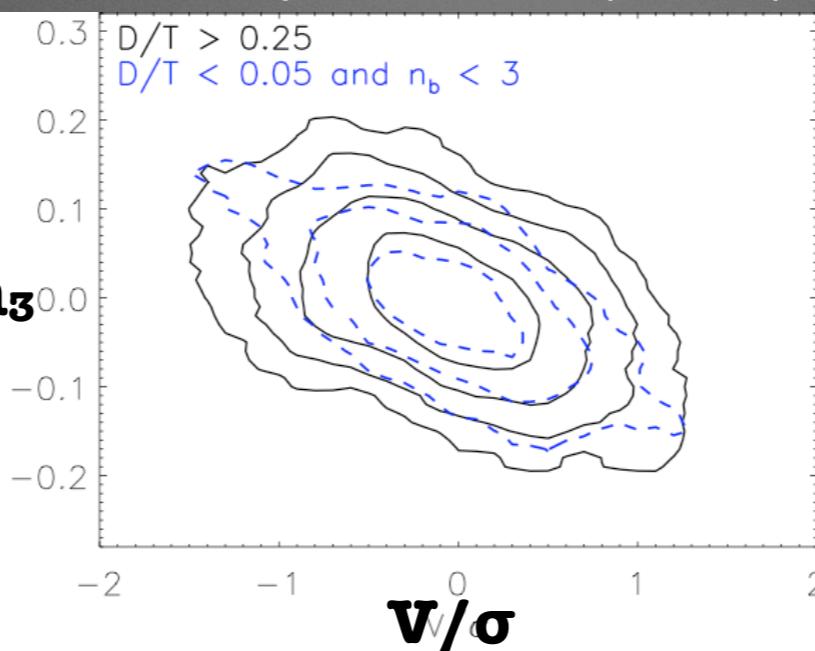


# Kinematic evidence for disks

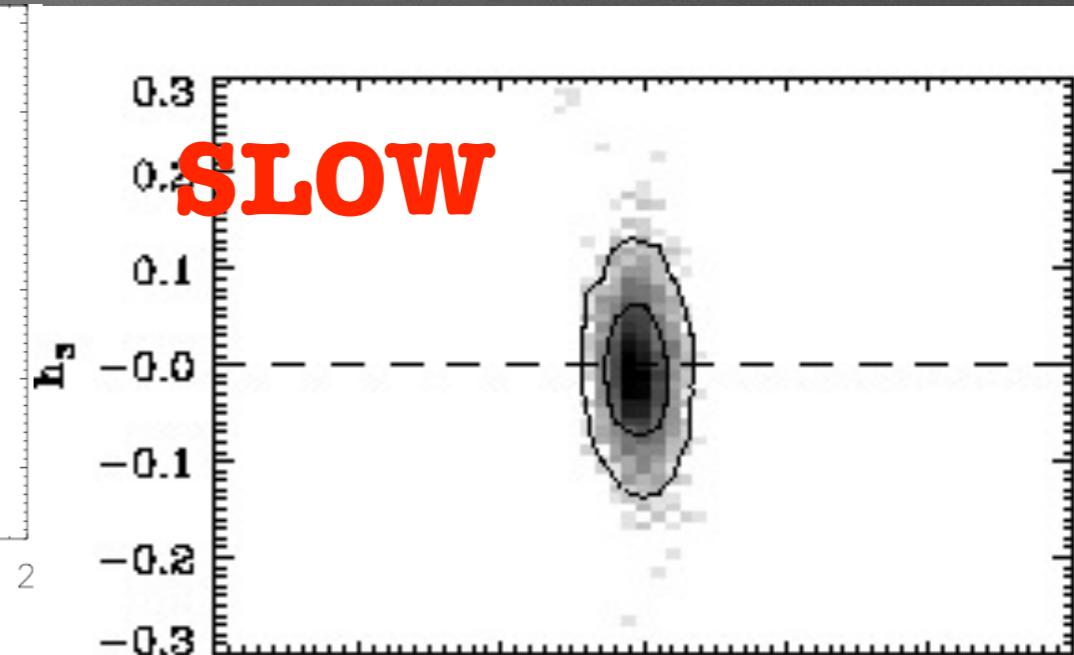
Bender et al. (1994)



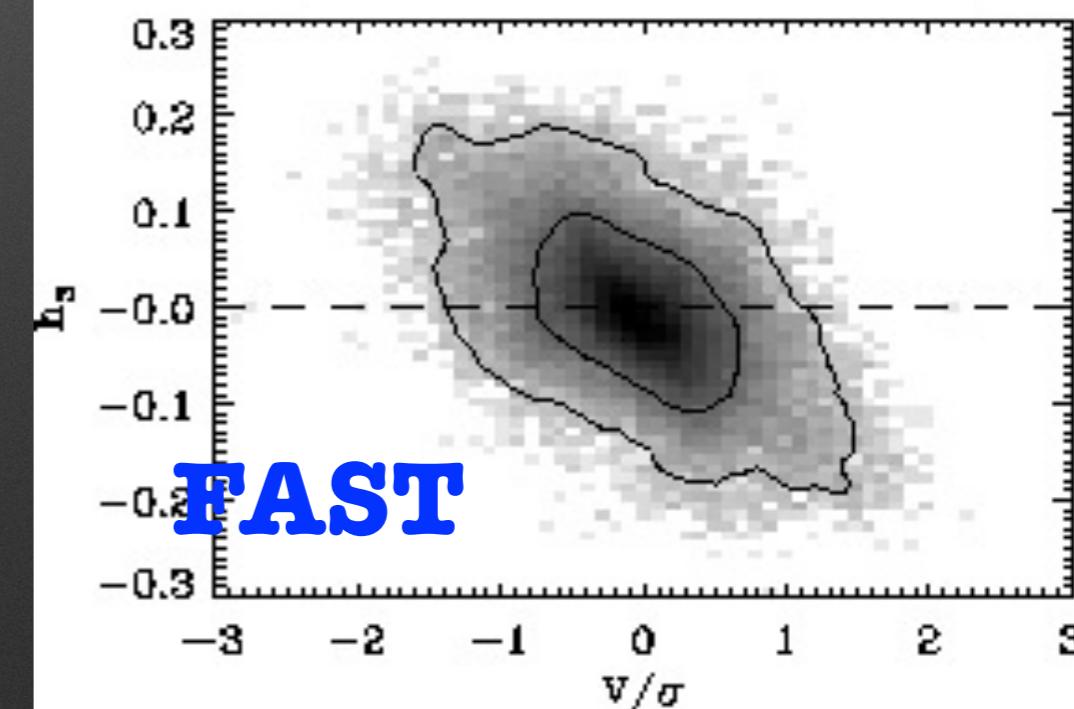
Krajnović et al. (2013a)



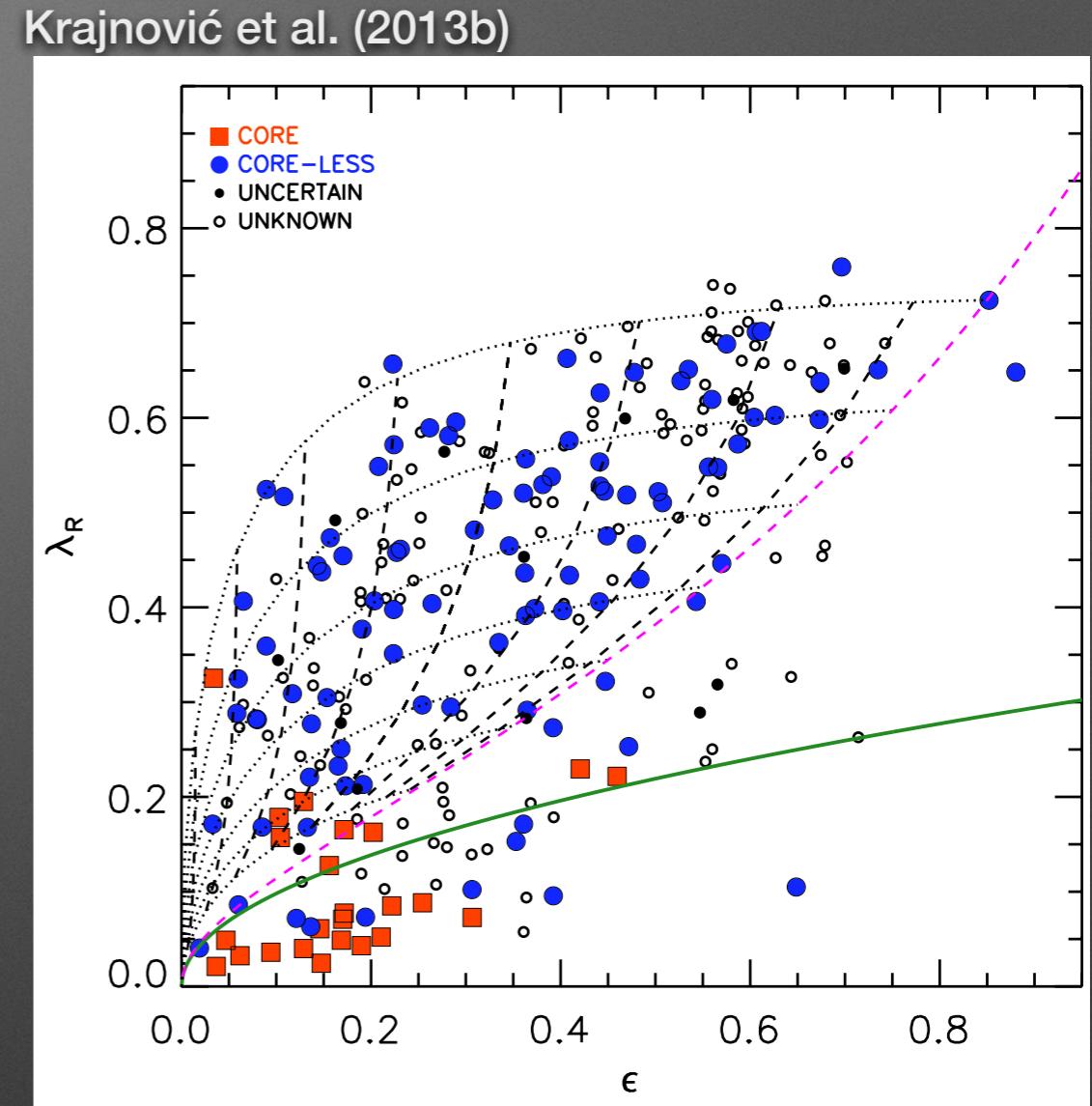
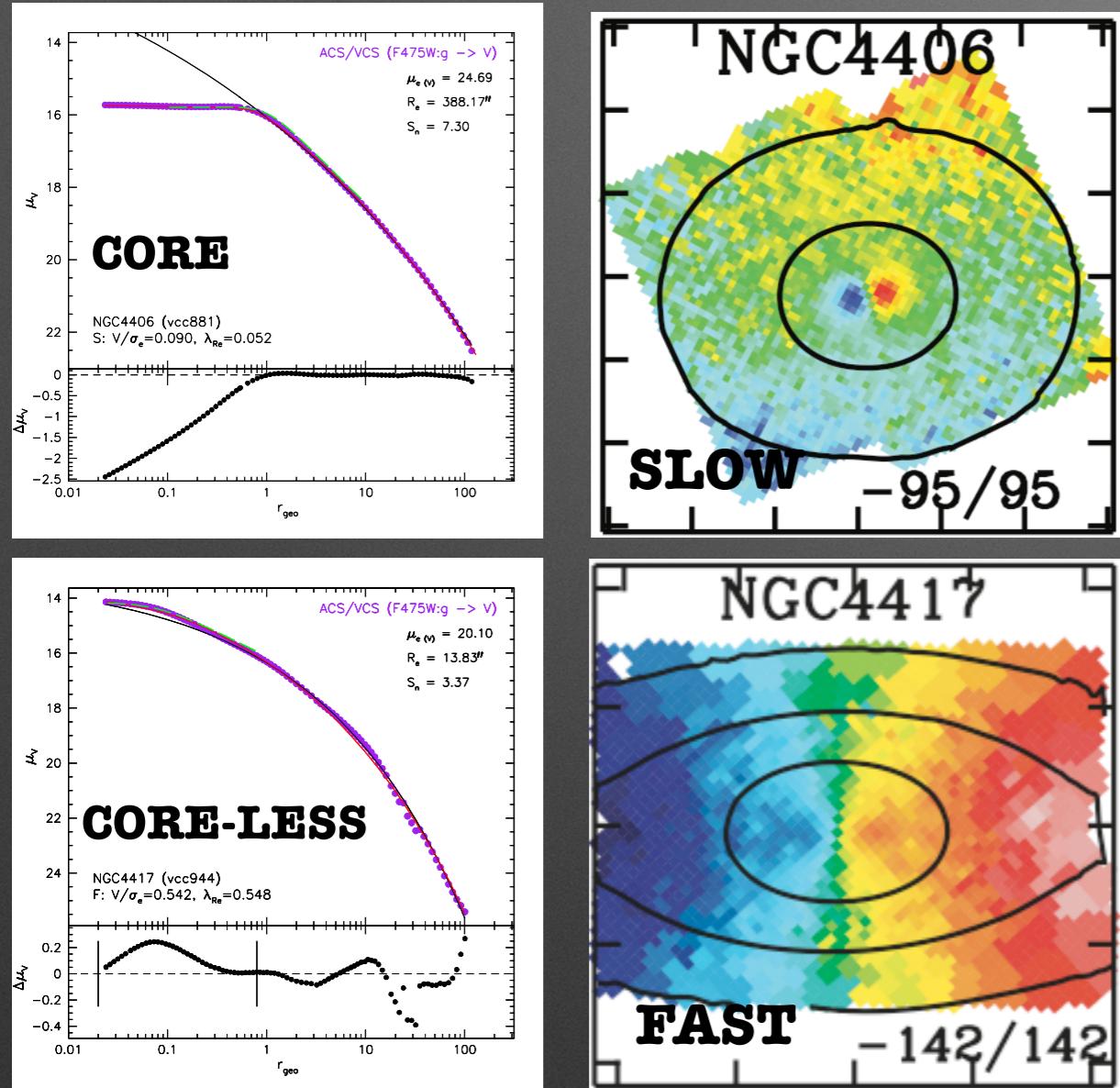
Krajnović et al. (2011)



- Asymmetric deviation measured in LOSVD by  $h_3$  Gauss-Hermite coefficient (van der Marel & Franx 1993, Gerhard et al. 1993)
- Typical of embedded disks
- Fast rotators – have disks
- Slow rotators – no disks

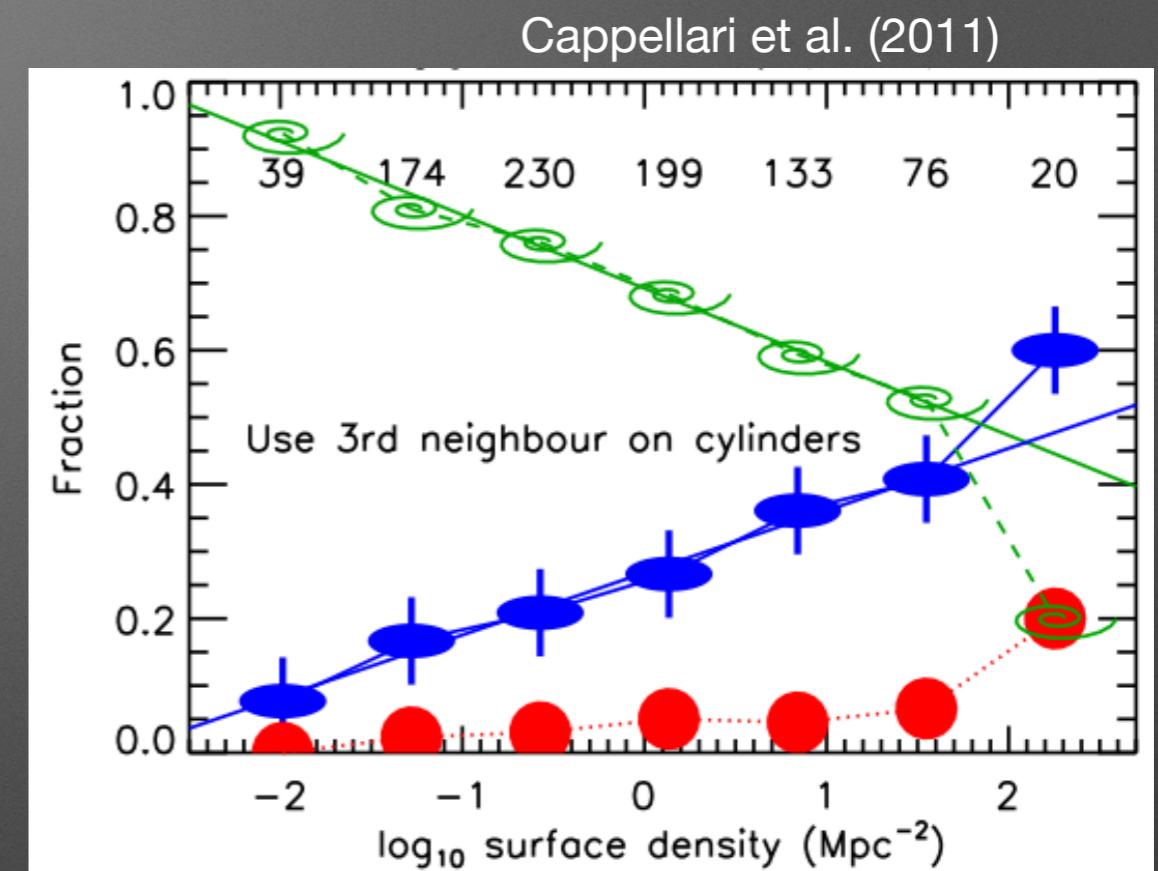
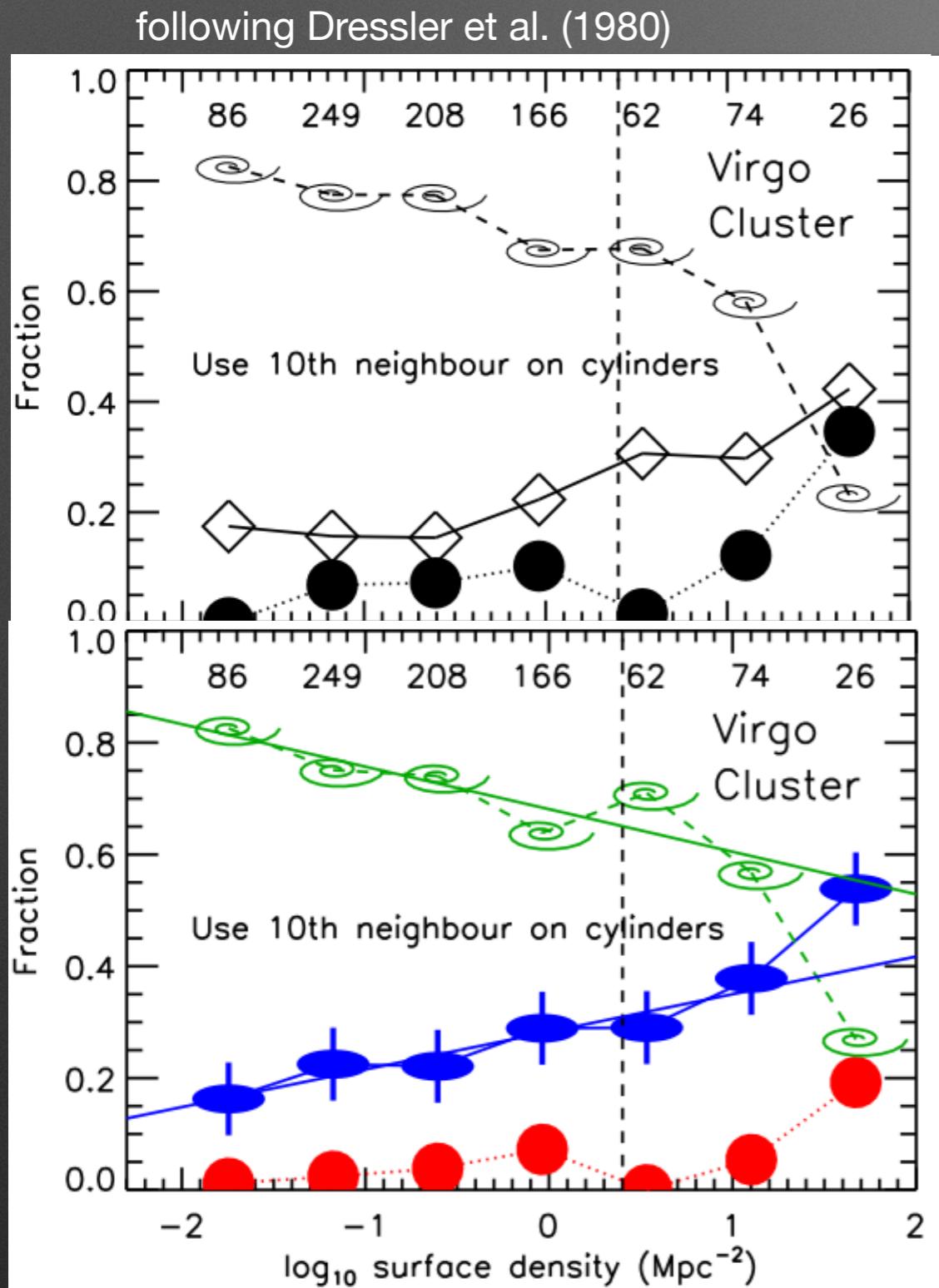


# Nuclear surface brightness profiles



- core galaxies are typically Slow Rotators, but not all
- most massive (lowest angular momentum) Slow Rotators have cores
- how does one make fast rotators with cores?

# Kinematic morphology - density relation



More local density measure:

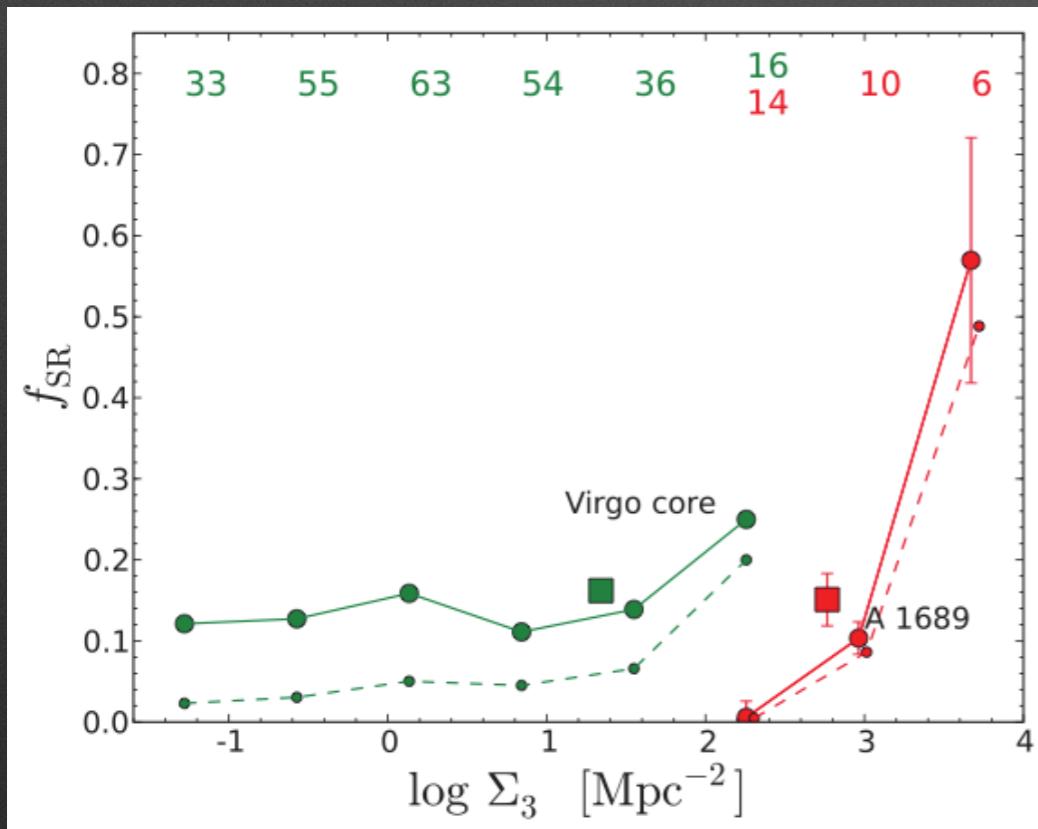
$$f(\text{Sp}) = 0.69 - 0.11 \times \log \Sigma_3$$

$$f(\text{FR}) = 0.26 + 0.09 \times \log \Sigma_3$$

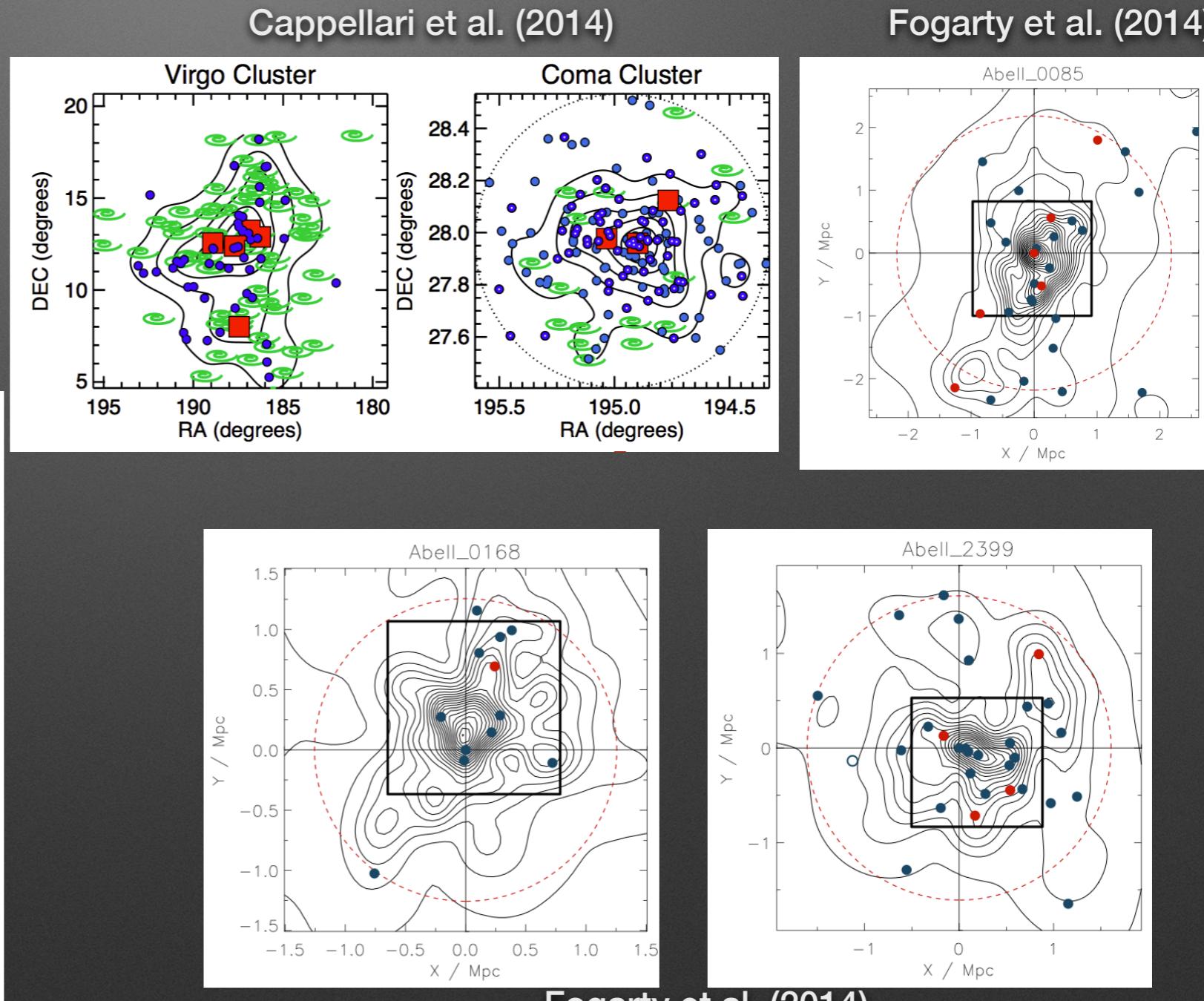
Transformation Spirals  $\rightarrow$  FR  
SR don't participate except at highest densities

# Kinematic morphology - density relation

- Slow rotators live in cores of clusters
- but also in groups

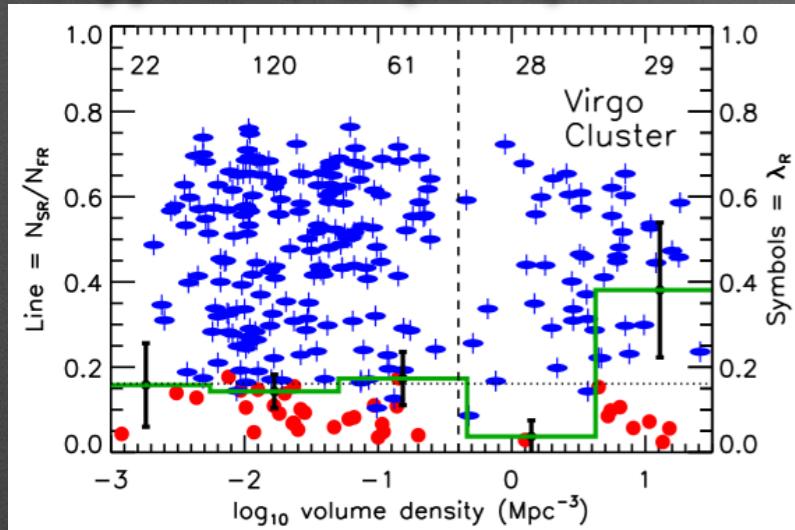


d'Eugenio et al. (2014)

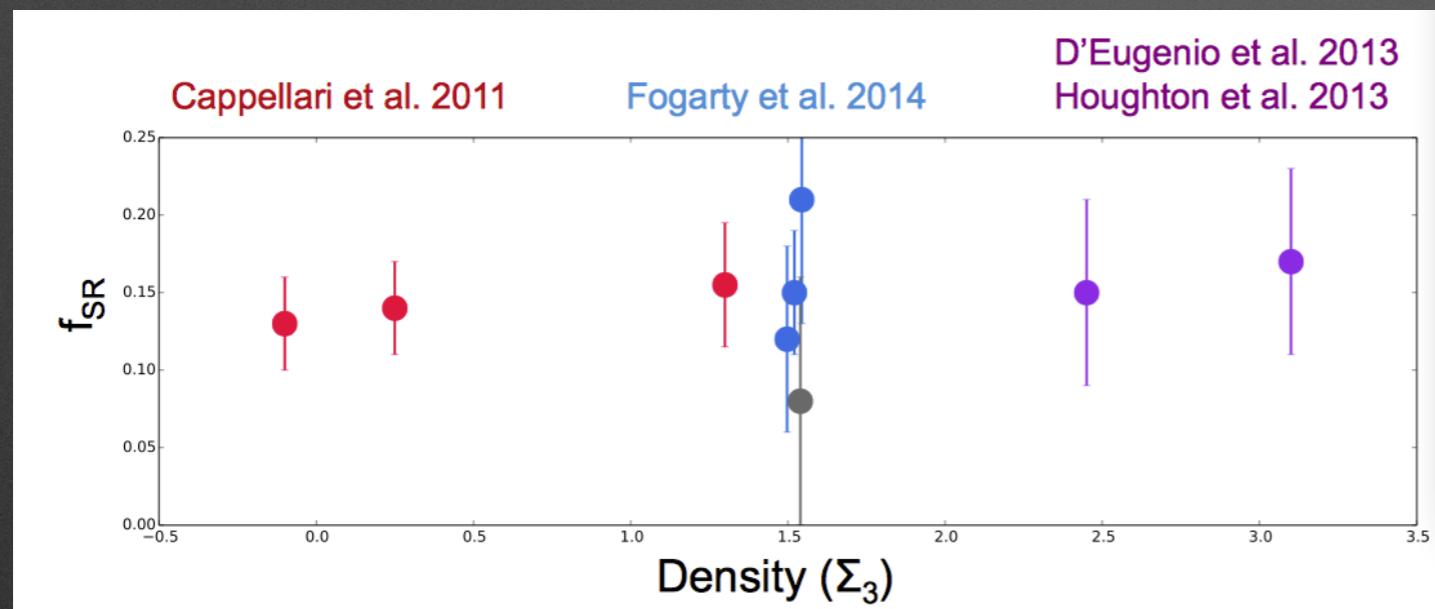
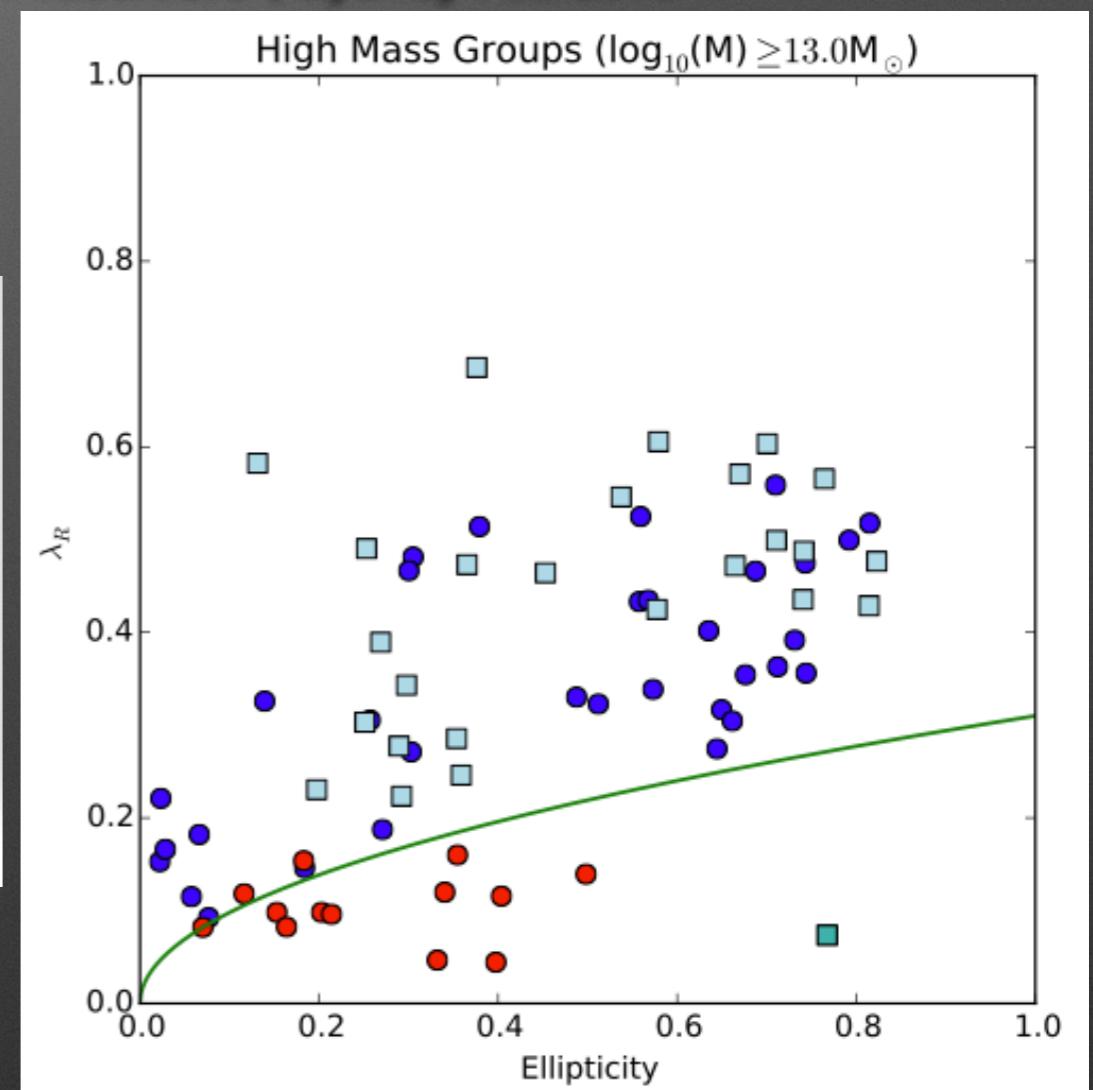


# Making slow rotators

Cappellari et al. (2011b)

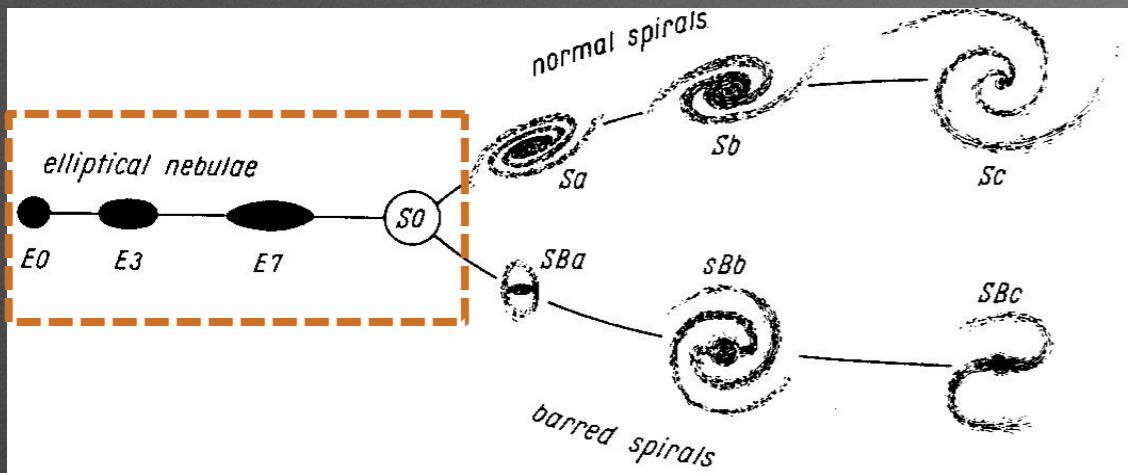


From Lisa Fogarty's talk at "Most Massive Galaxies and their Precursors", Sydney Feb 2015

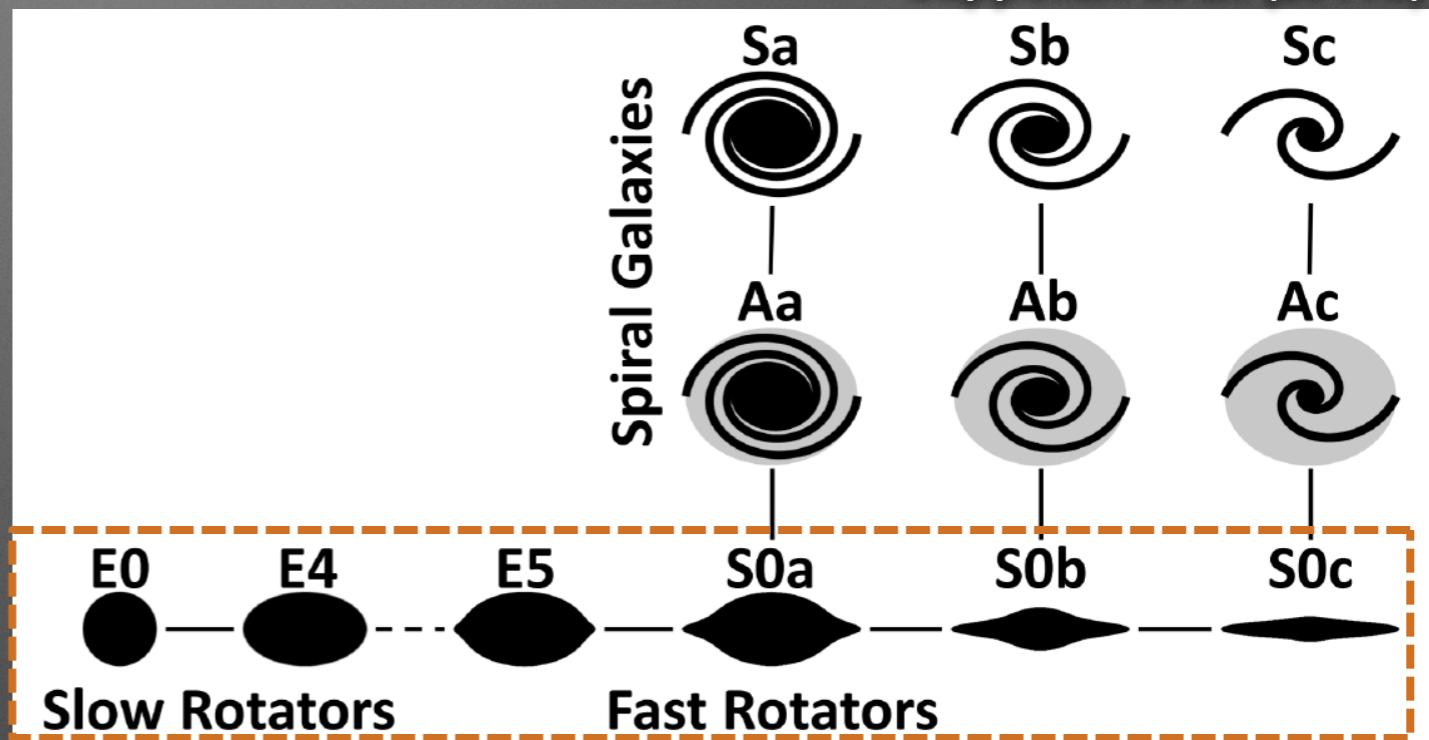


- global efficiency of producing SR is constant
- (massive) groups as sites of formation (?) (Fogarty et al. 2014)
- see also Cappellari et al. (2011), Houghton et al. (2013), D'Eugenio et al. (2014)

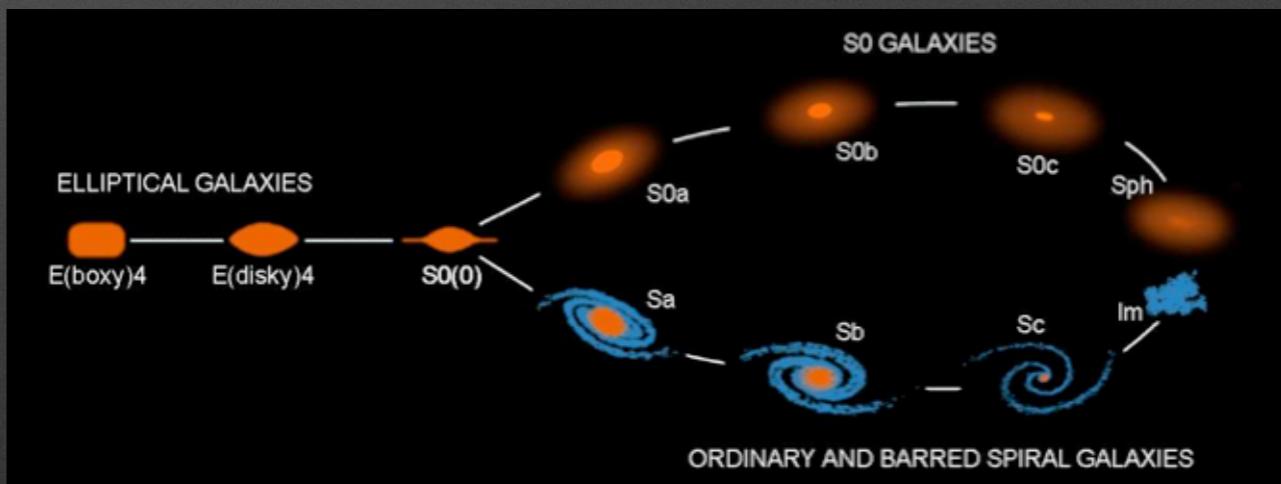
# Galaxy classification (learning from kinematics)



Hubble (1935)



- Fast rotators: parallel sequence to spirals (see Spitzer & Baade 1951; van den Bergh 1976)
- ETGs generally not spheroidal but disk-like
- Hubble diagram (in form of Kormendy & Bender 1996) does not properly describe ETGs—> needs updating
- Laurikainen et al. (2011), Cappallari et al. (2011b), Kormendy & Bender (2012)



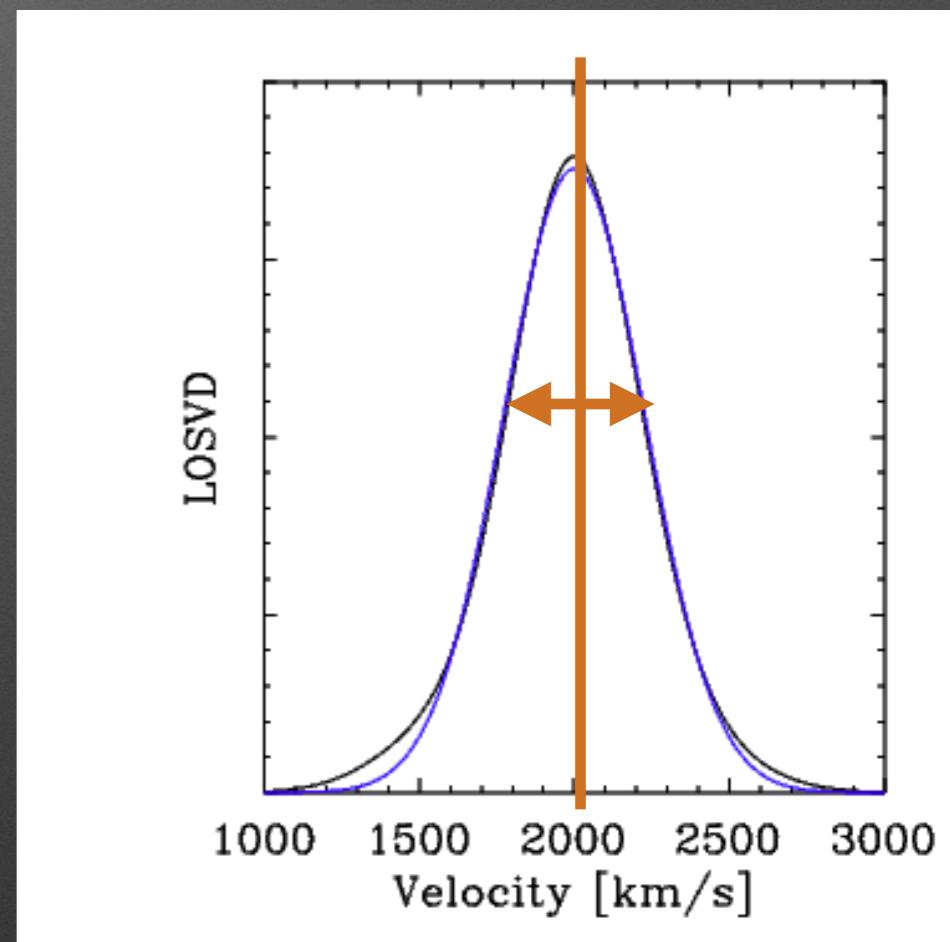
Kormendy & Bender et al. (2012)

# Dynamics of galaxies

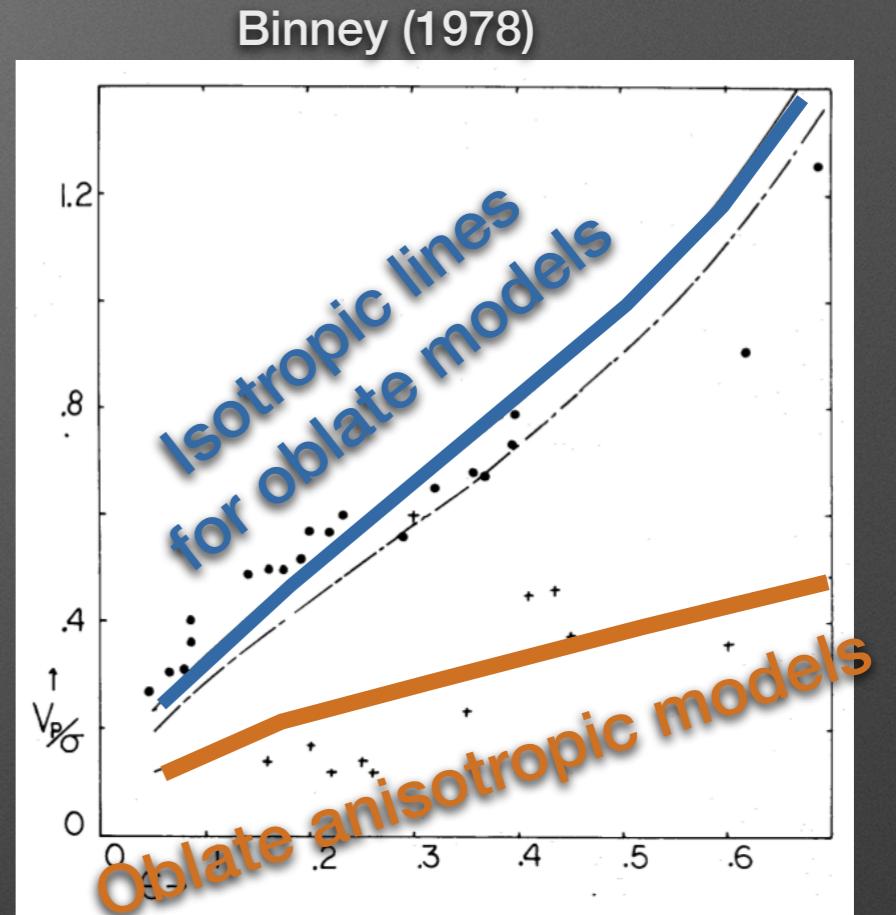
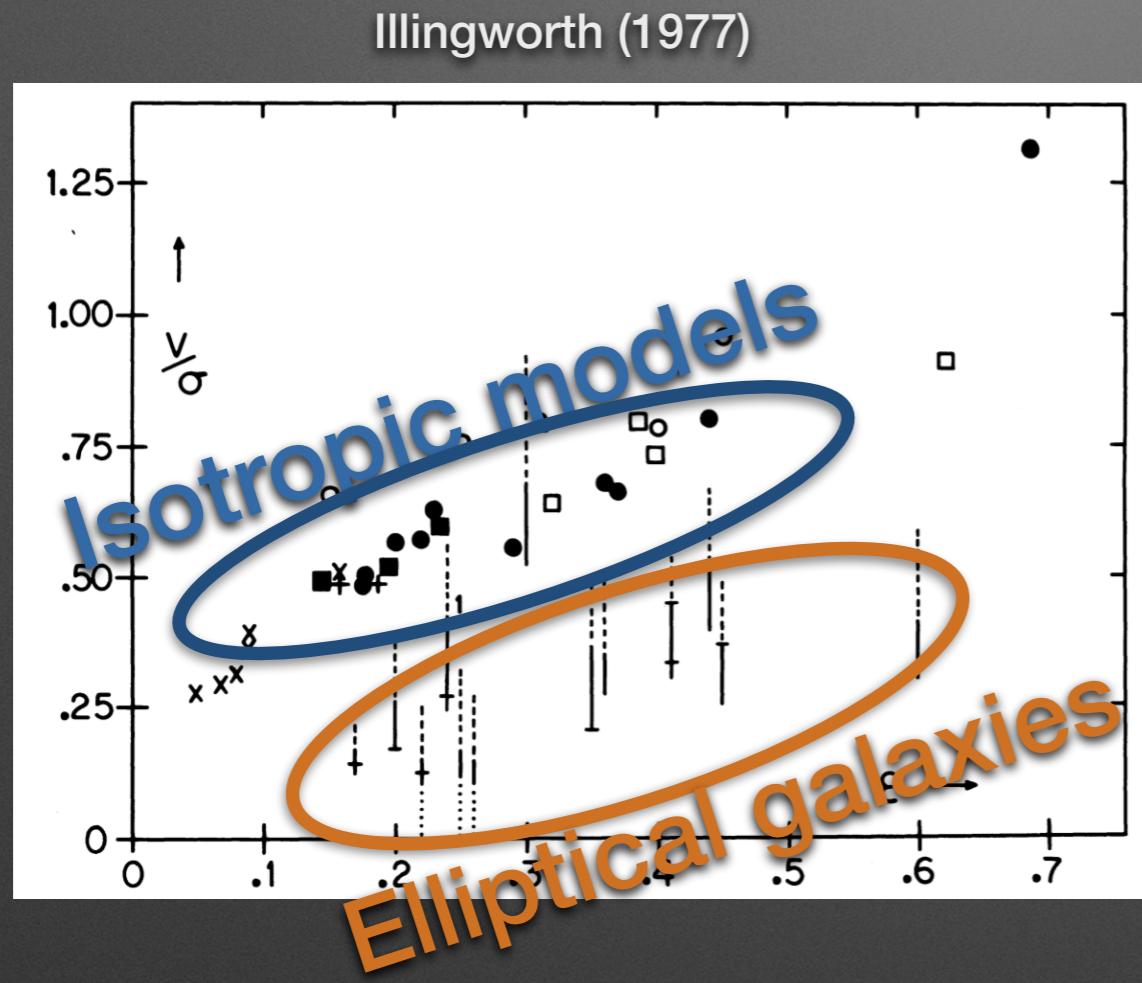
# Preliminaries

- Components of the LOSVD:
  - mean:  $v = \int dv_{\text{los}} v_{\text{los}} \text{LOSVD}(x, v)$ 
    - mean streaming of stars
  - dispersion:  $\sigma^2 = \int dv_{\text{los}} (v_{\text{los}} - v)^2 \text{LOSVD}(x, v)$ 
    - random motion of stars
  - ratio of ordered to random motion is related to the shape
  - no rotation  $\rightarrow$  spherical shape
  - rotation  $\rightarrow$  flattened shape
  - if random motion is same in all directions (oblate isotropic rotators (Illingworth 1977)
    - $V/\sigma \approx \sqrt{\epsilon/(1-\epsilon)}$  (Binney 1978, Kormendy 1982, Binney & Tremaine 2008)

depends on the mean streaming and spread in stellar velocities at each point



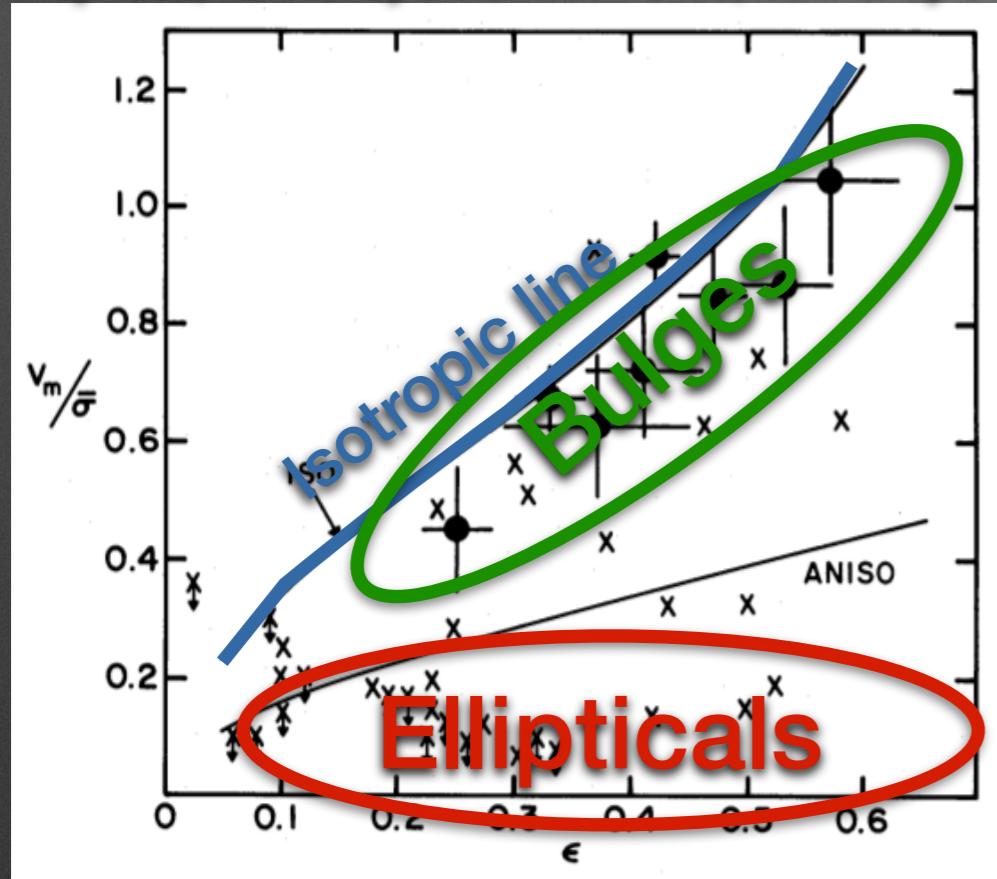
# Introducing the $(V/\sigma, \epsilon)$ diagram



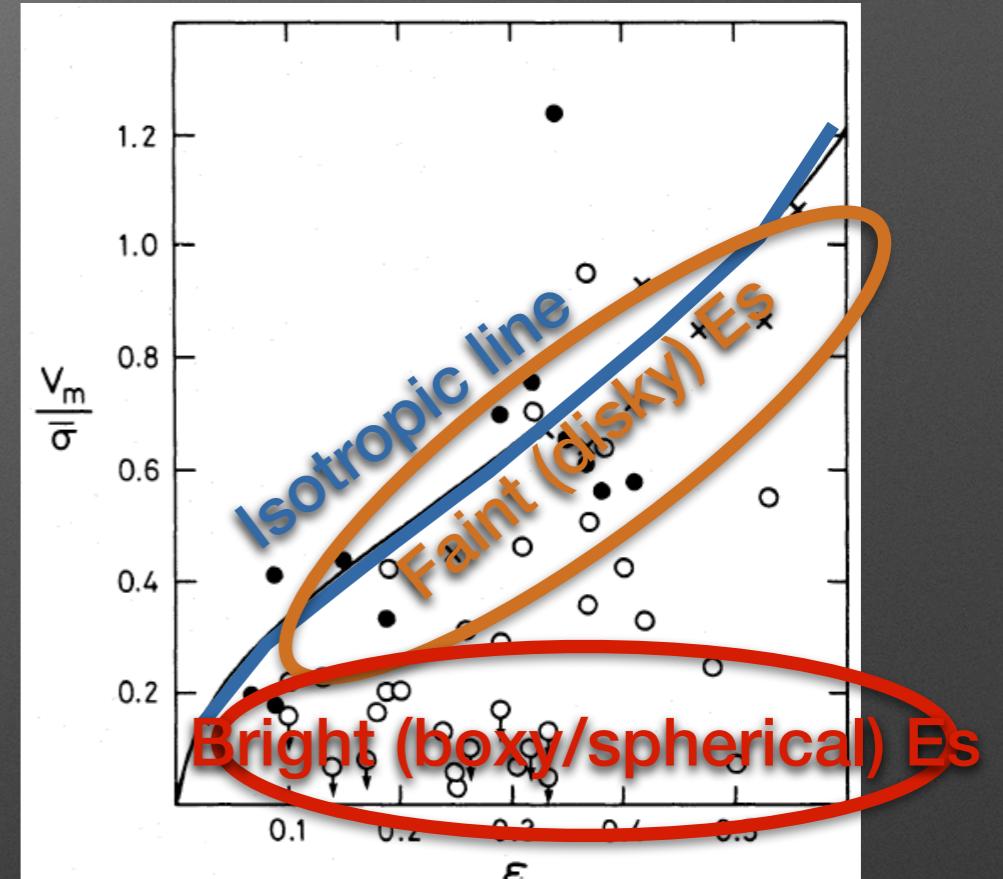
- Illingworth:  $V/\sigma$  for first elliptical galaxies
  - less rotation than expected for isotropic systems
- Binney: linking tensor virial theorem to observations

# The power of ( $V/\sigma$ , $\epsilon$ ) diagram

Kormendy & Illingworth (1982)  
[Kormendy 1982, Kormendy & Kennicutt 2005, Kormendy & Fisher (2008)]



Davies et al. (1983)  
(Bender 1988), Kormendy & Bender (1996)

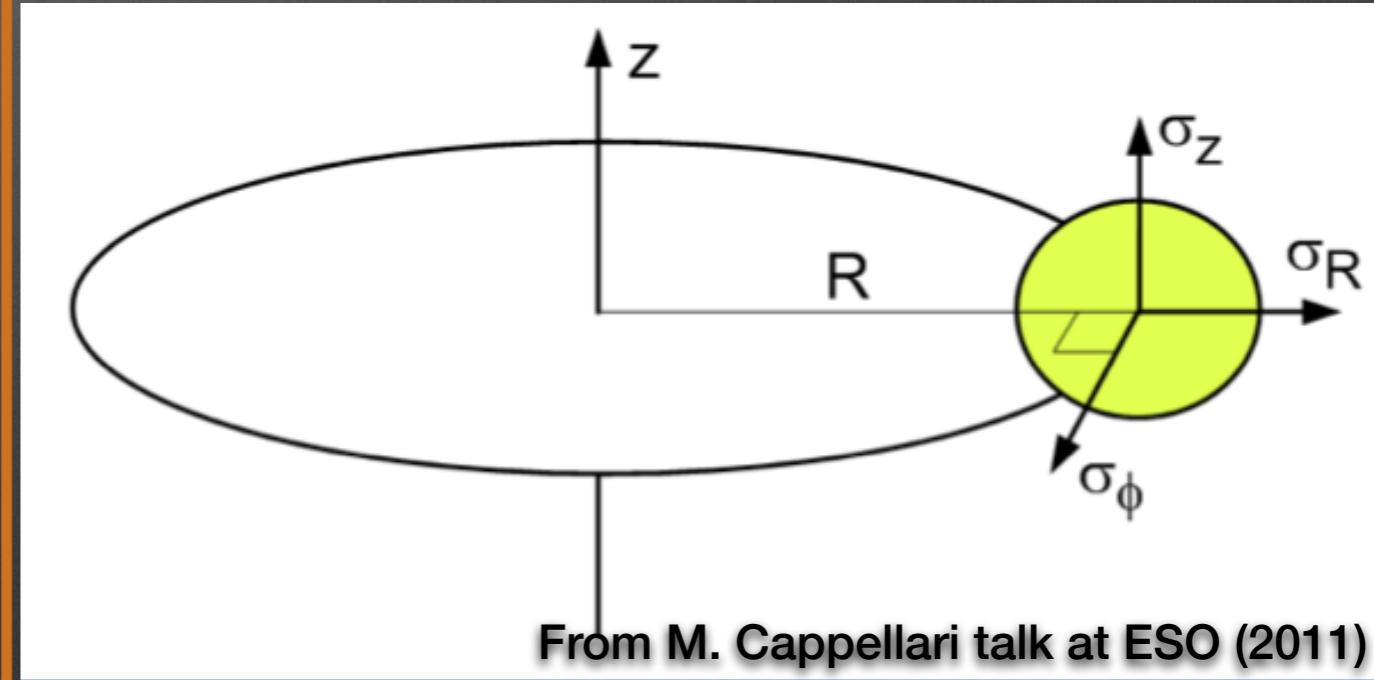


- Bulges close to isotropic line
- faint ellipticals similar to bulges
- bright ellipticals rotate slowly
- high  $V/\sigma$ : fainter, disk-like ellipticals
- low  $V/\sigma$ : bright boxy ellipticals

# Velocity ellipsoid & anisotropy

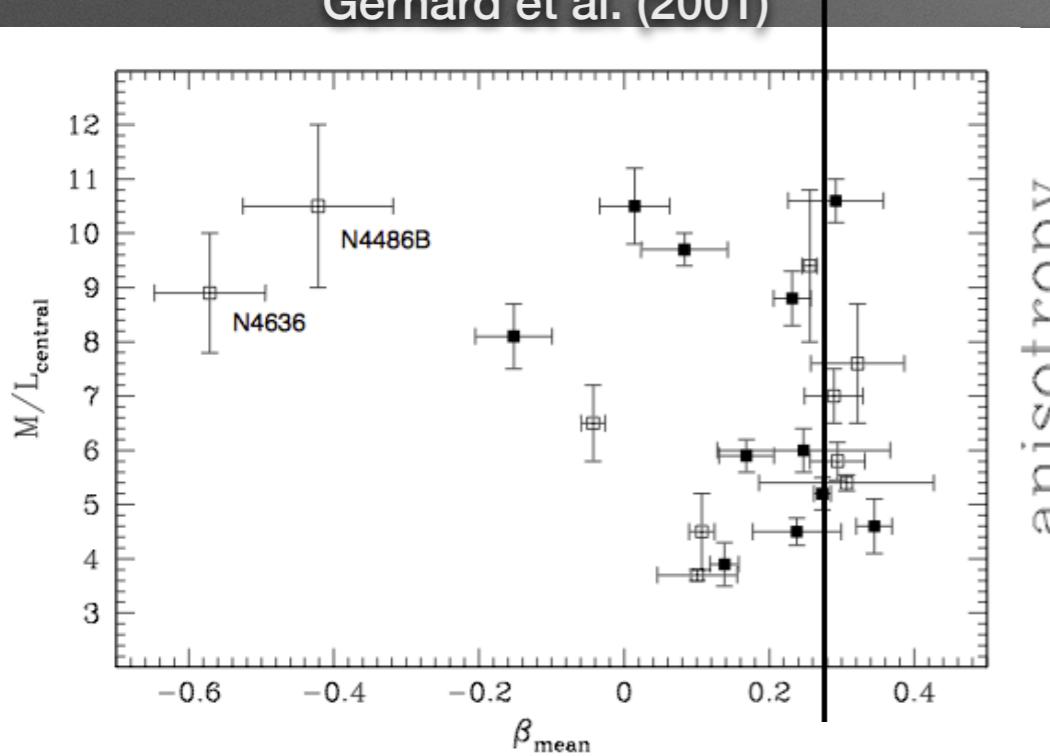
- velocity dispersion tensor:
  - $\sigma_{ij}^2 = 1/v \int d^3v (v_i - \langle v_i \rangle)(v_j - \langle v_j \rangle)$  LOSVD(x,y) =  $\langle v_i v_j \rangle - \langle v_i \rangle \langle v_j \rangle$
- velocity ellipsoid defined in set of orthogonal axes in which  $\sigma^2$  is diagonal

- $\sigma_z = \sigma_\phi = \sigma_R$  (isotropy)
- $\beta = 1 - \sigma_z^2/\sigma_R^2$
- $\gamma = 1 - \sigma_\phi^2/\sigma_R^2$
- $\delta = 1 - 2\sigma_z^2/(\sigma_R^2 + \sigma_\phi^2)$
- $\sigma_\phi^2 = \sigma_R^2 \rightarrow (\delta = \beta, \gamma = 0)$ :  
oblate velocity ellipsoid
- $\delta$  (from  $V/\sigma, \epsilon$ );
- $\beta$  &  $\gamma$  need dynamical models

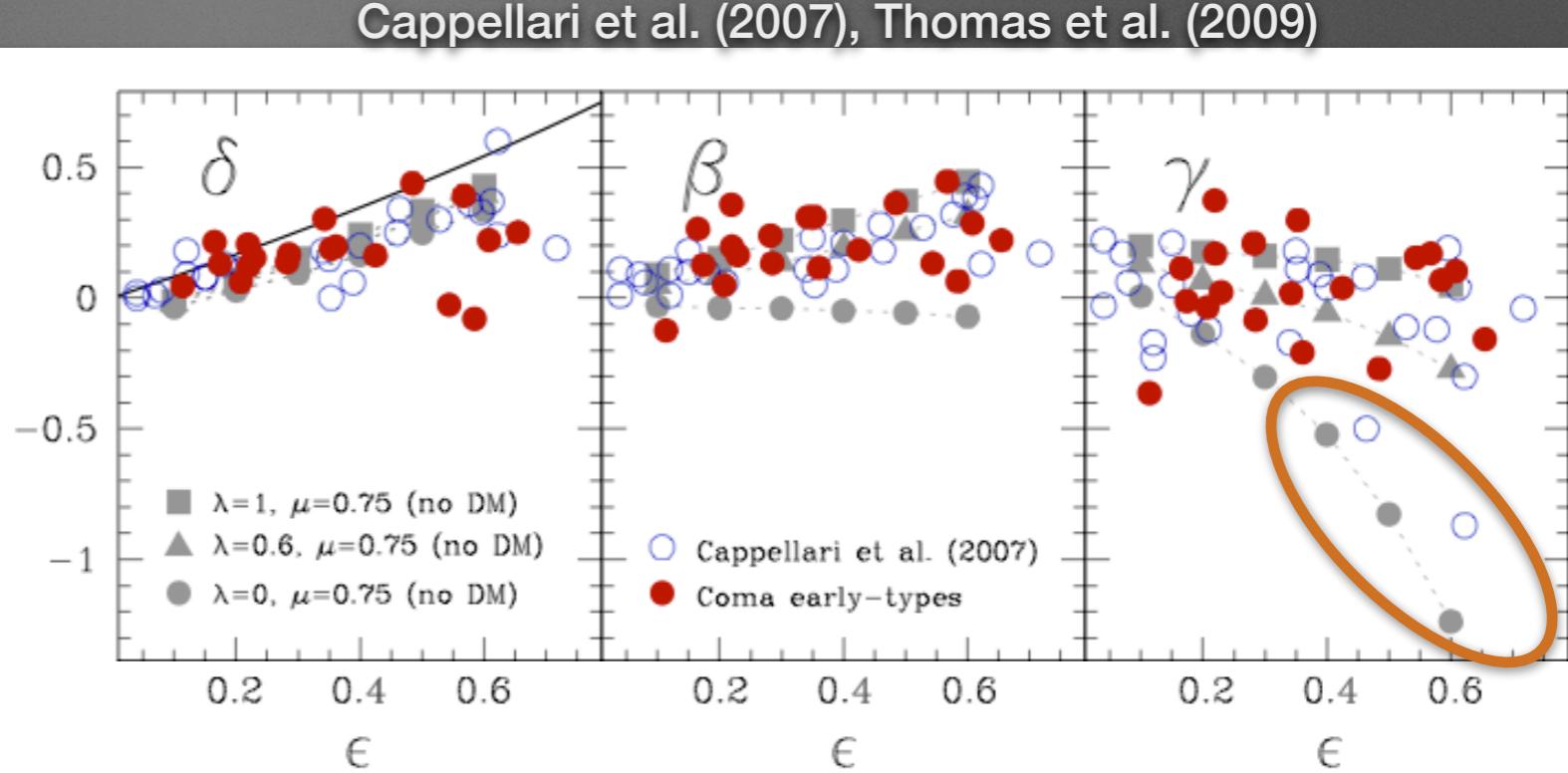


# What anisotropy is observed?

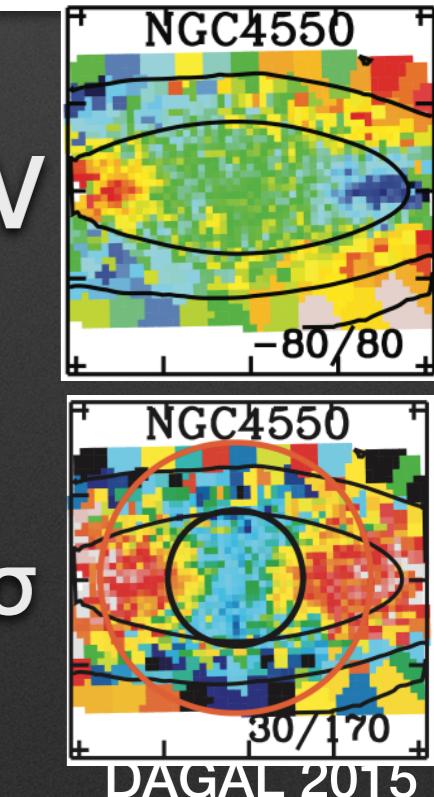
Gerhard et al. (2001)



Cappellari et al. (2007), Thomas et al. (2009)



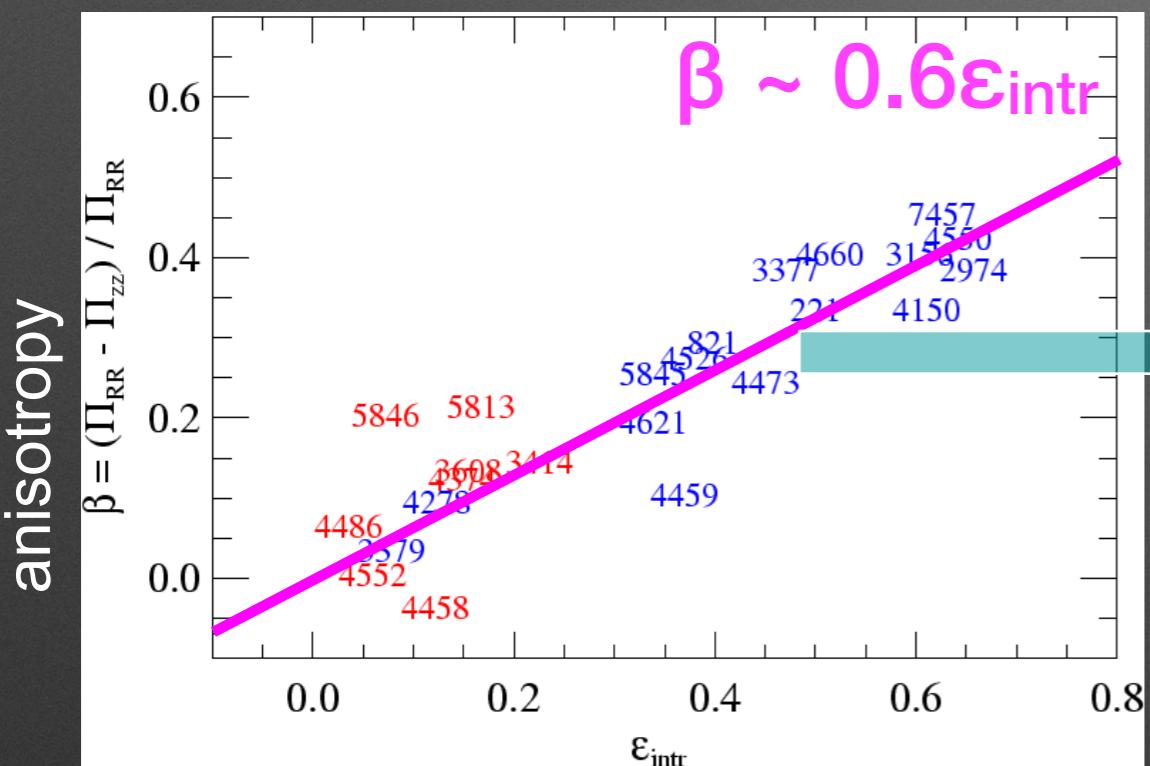
- spherical dynamical models (of giant Es):  $\beta \sim 0.25$
- axisymmetric (Schwarzschild) models (of flattened ETGs):
  - $\beta \approx \delta$  and  $\gamma \sim 0$  (oblate velocity ellipsoid)
  - anisotropy is proportional to (intrinsic) shape of galaxies  
 $\beta \sim 0.6\epsilon_{\text{intr}}$  (Cappellari et al. 2007)



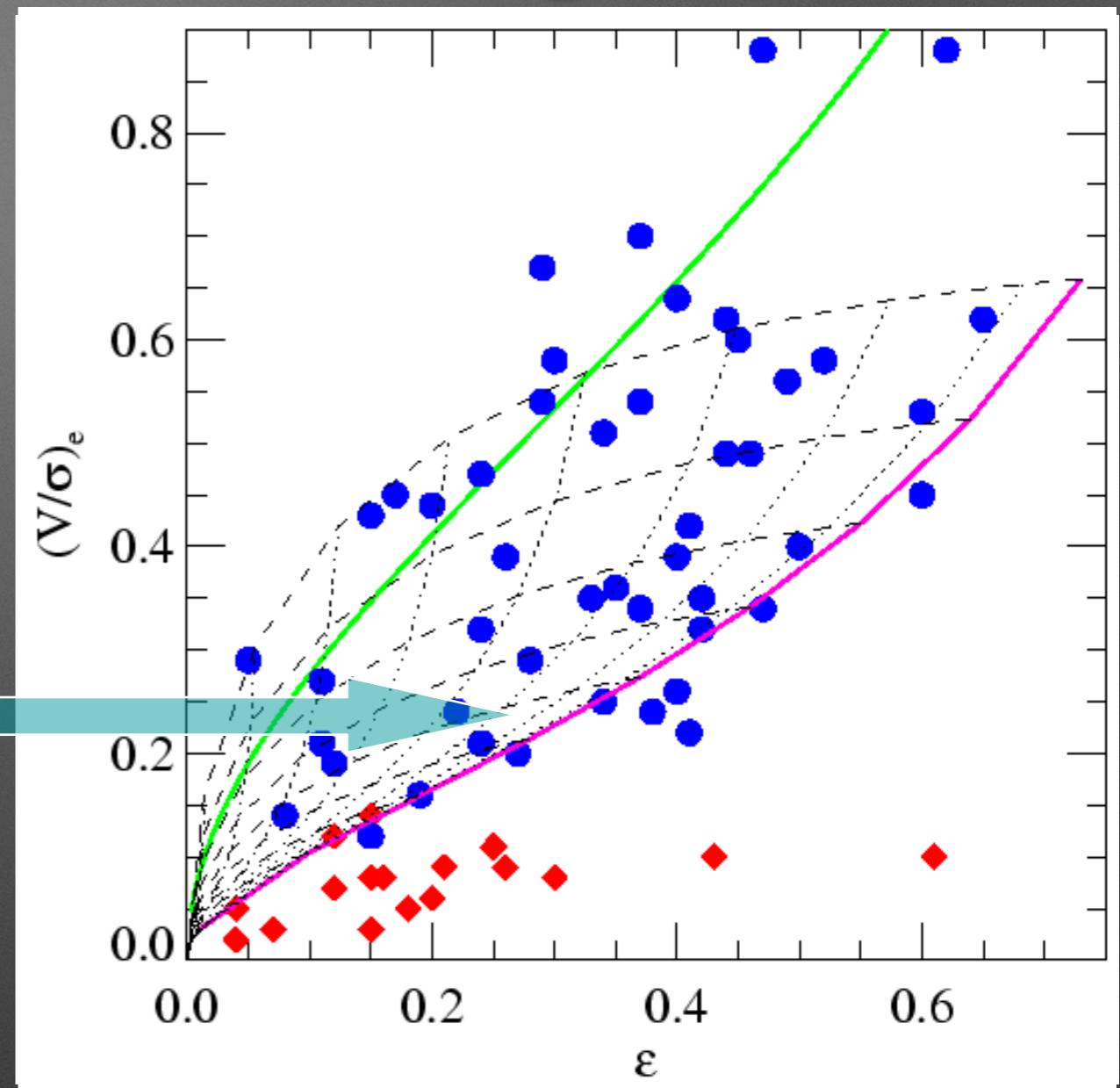
# Revisiting the V/ $\sigma$ diagram

$$(V/\sigma)_e^2 \equiv \frac{\langle V^2 \rangle}{\langle \sigma^2 \rangle}$$

Use new formalism for integral-field kinematics  
(Binney 2005)



Anisotropy trend from 24 Models (Cappellari et al. 2007)



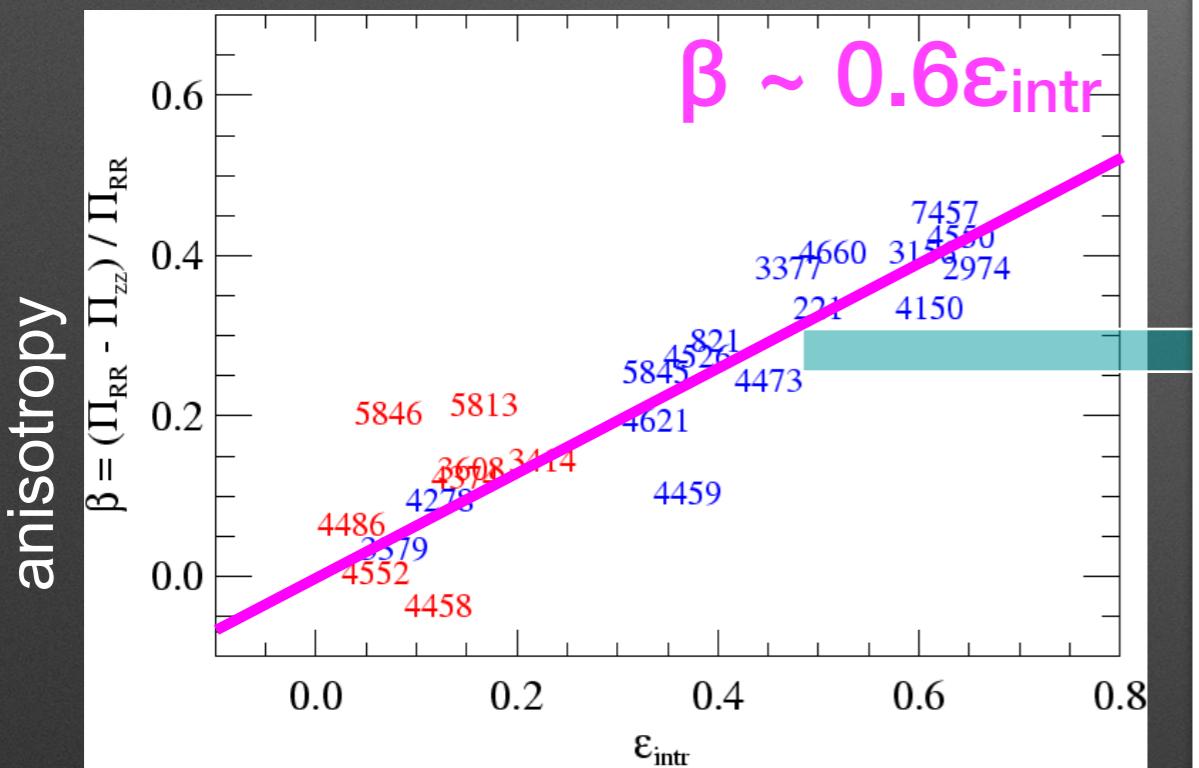
SAURON sample of 66 Cappellari et al. (2007)

- **Fast-rotators:** family of oblate systems
- **Slow-rotators:** distinct - likely triaxial
- **ATLAS3D sample:** statistics (but new kids on the block: CALIFA, SAMI, MANGA)

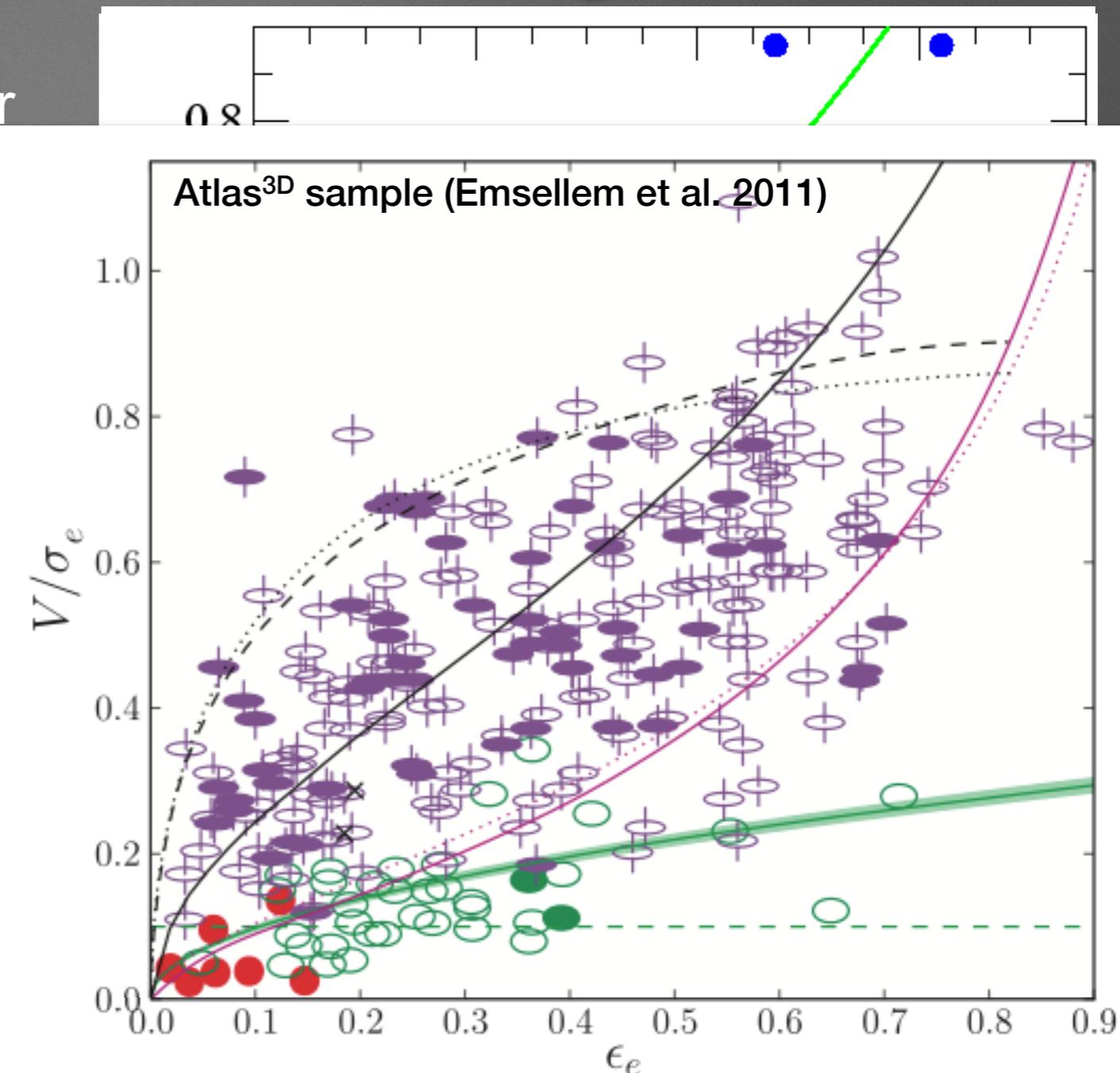
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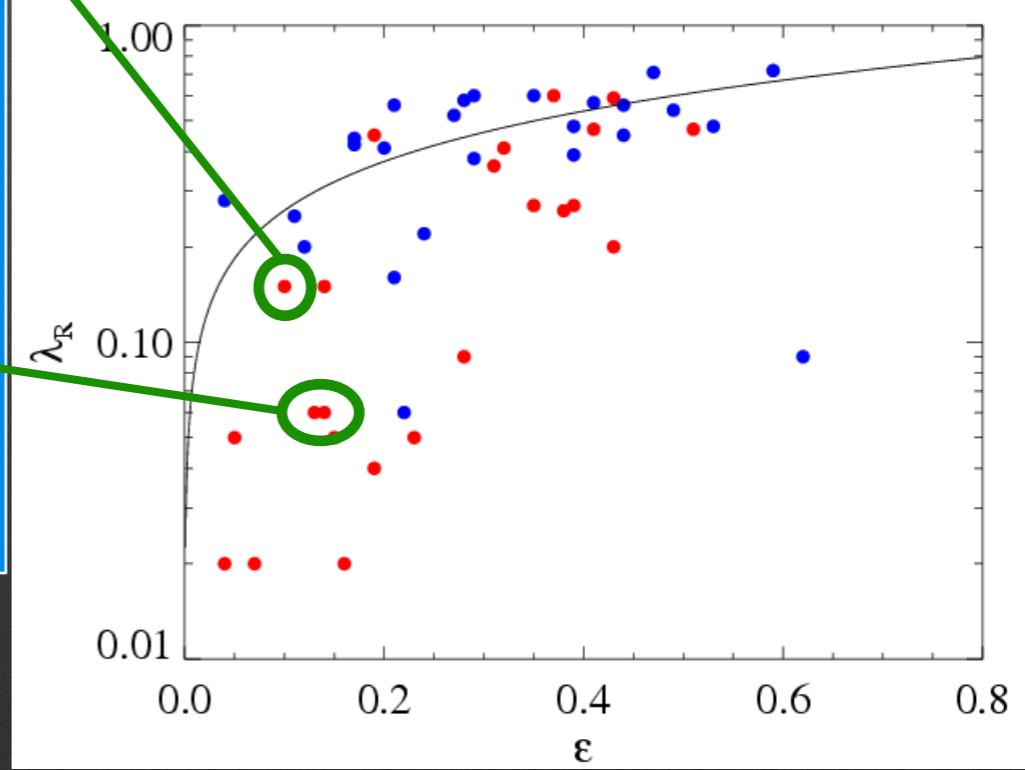
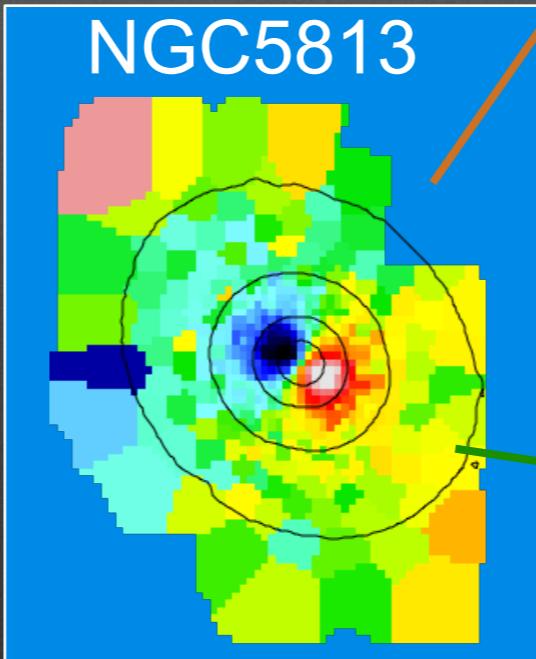
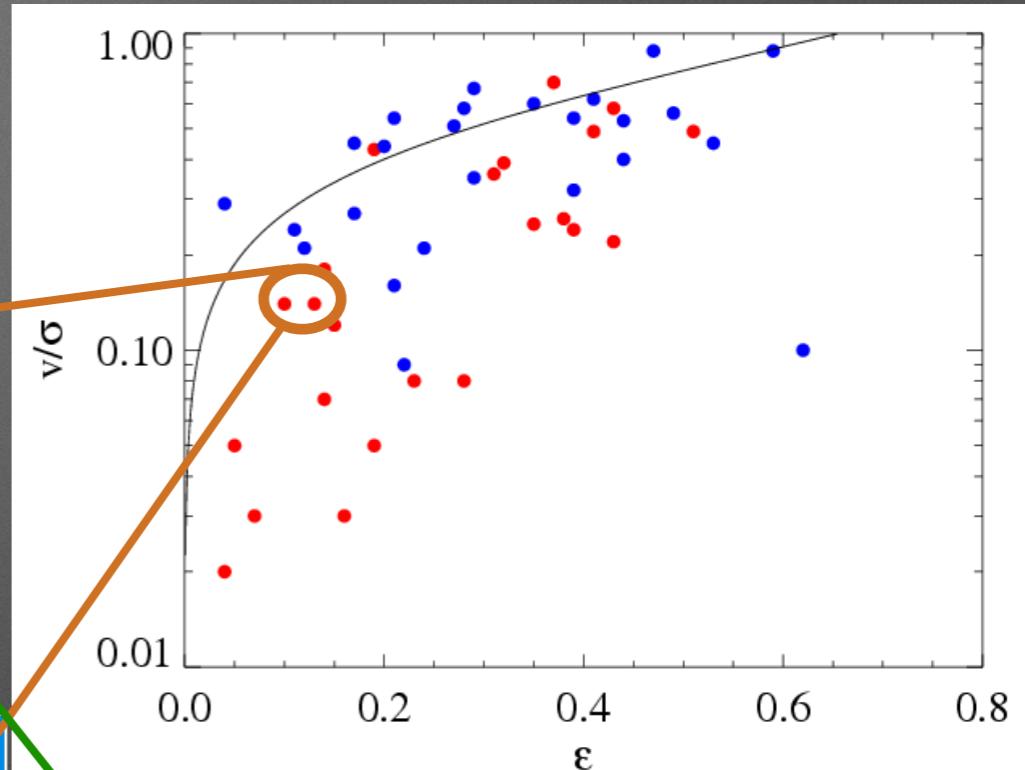
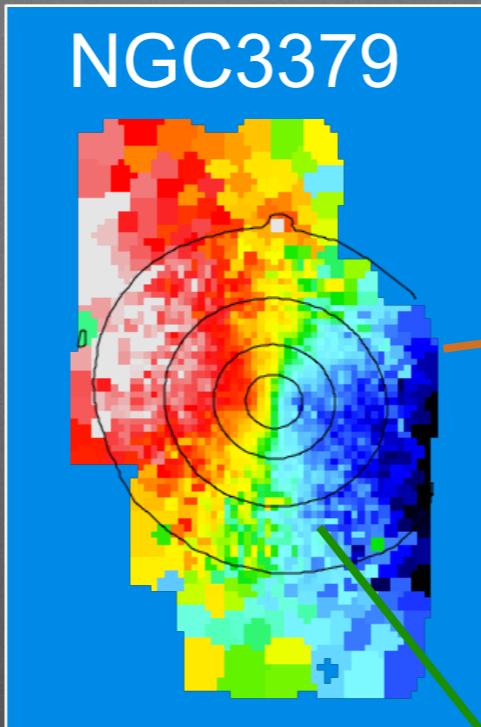
- **Fast-rotators:** family of oblate systems
- **Slow-rotators:** distinct - likely triaxial
- **ATLAS3D sample:** statistics (but new kids on the block: CALIFA, SAMI, MANGA)

# V/ $\sigma$ or $\lambda$ ?

- which diagram is better?
  - depends what you want to do
  - linked:

$$\lambda_R = \frac{\langle RV \rangle}{\langle R\sqrt{V^2 + \sigma^2} \rangle} \approx \frac{\kappa (V/\sigma)}{\sqrt{1 + \kappa^2 (V/\sigma)^2}}$$

- rotational characteristics better separated in  $(\lambda, \varepsilon)$  diagram



# Dynamical models

# Simple mass estimates

- mass distribution expected to be dominated by baryons in the central regions ( $1-2 R_e$ ) and DM in the outskirts
- (scalar) Virial theorem (Clausius 1870):  $2K + W = 0$
- relation between the kinetic energy and gravitational potential energy in a steady state system (in isolation)
- problem: what do we observe?
  - can not ‘integrate’ over the entire galaxy!!
  - ‘effective’ radius:  $R_e$  ( $R_{1/2} = 1.33R_e$ )
  - ‘effective’ velocity dispersion:  $\sigma_e$
  - ‘effective’ mass (-to-light ratio)

BT 08

$$M = \frac{r_g \langle v^2 \rangle_\infty}{G}$$

total mass  
(luminous + dark)

gravitational  
radius

mean square  
speed of stars,  
integrated over  
the full extent  
of the system

$$M_{1/2} = k \frac{r_{1/2} \langle \sigma_{\text{los}}^2 \rangle_\infty}{G}$$

half light  
(spherical)  
radius

Cappellari et al. (2006)

$$(M/L)(r = R_e) \approx 5.0 \times \frac{R_e \sigma_e^2}{GL}$$

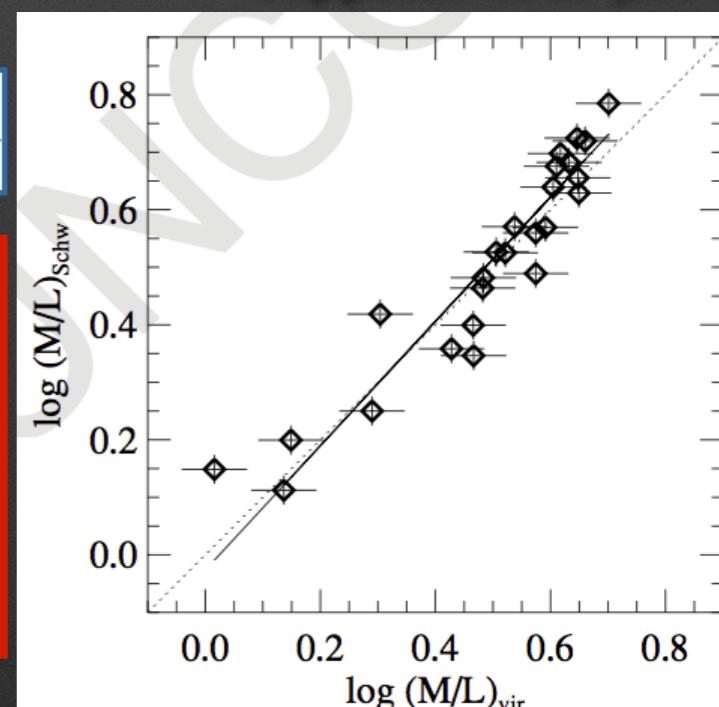
velocity  
dispersion,  
integrated over  
the full extent  
of the system

Cappellari et al. (2013)

$$M_{1/2} \approx 2.5 \times \frac{R_e \sigma_e^2}{G} \approx 1.9 \times \frac{r_{1/2} \sigma_e^2}{G}$$

- 1) measuring  $\sigma_e$ : spectrum within an (circular/elliptical) aperture of radius  $R_e$
- 2) extract kinematics (Gaussian LOSVD)

Cappellari et al. (2006)



# Collision-less Boltzmann equation

- galaxy: collisionless stellar system
  - for inner regions: two-body relaxation time-scale  $\gg$  dynamical time-scale
- Distribution function:  $f(\vec{x}, \vec{v}, t)$ 
  - must  $f > 0$ , must satisfy continuity equation in  $(\vec{x}, \vec{v})$
  - Collisionless Boltzmann Equation (CBE)
  - simultaneous solution to these equations (self-consistent problem)
  - 1) assume  $\rho$ , calculate  $\Phi$ , get  $f$
  - 2) assume  $f$ , integrate to get  $\rho$  and solve for  $\Phi$

$$\frac{\partial f(\vec{x}, \vec{v}, t)}{\partial t} + \vec{v} \cdot \nabla f(\vec{x}, \vec{v}, t) - \nabla \Phi(\vec{x}, t) \frac{\partial f(\vec{x}, \vec{v}, t)}{\partial \vec{v}} = 0$$

$$\nabla^2 \Phi(\vec{x}, t) = 4\pi G \rho(\vec{x}, t)$$

$$\rho = \int \int \int dv_x dv_y dv_z f(\vec{x}, \vec{v})$$

Force per unit mass

Total gravitational potential

Total mass density

- solving CBE is hard ( $f$  - 6 dimension,  $\rho$  hard to probe)
- looking at local variable (not directly looking at  $f$ )
- looking into the first velocity moment of CBE

# Jeans equation

- Euler's equation for a fluid ("stellar hydrodynamics")
- anisotropic stress tensor  $v\sigma^2$
- $v$  - density of the tracer: it can (but doesn't have to) be proportional to the total mass density  $\rho$
- instead of  $f$ , looking into 1st two velocity moments of the LOSVD
- problems:
  - no equation of state between pressure and density (system is not closed)!
  - no guaranty that  $f \geq 0$  (everywhere)

Jeans equation  
multiply CBE by  $v$ , integrate over velocities

$$\frac{\partial \bar{\mathbf{v}}}{\partial t} + (\bar{\mathbf{v}} \cdot \nabla) \bar{\mathbf{v}} = -\nabla \Phi - \frac{1}{\nu} \nabla \cdot (\nu \sigma^2)$$

space density of the tracer

$$\nu = \int f d^3v$$

anisotropic (dynamical) pressure tensor

$$\begin{aligned}\sigma_{ij}^2 &= \bar{v}_i \bar{v}_j - \bar{v}_i \bar{v}_j \\ \bar{v}_i v_j &= \int v_i v_j f d^3v \\ \bar{v}_i &= \int v_i f d^3v\end{aligned}$$

velocity dispersion tensor

stationary Jeans equation  
no time dependance

$$\nu (\bar{\mathbf{v}} \cdot \nabla) \bar{\mathbf{v}} + \nabla \cdot (\nu \sigma^2) = -\nu \nabla \Phi$$

streaming

pressure

potential

# Spherical Jeans models

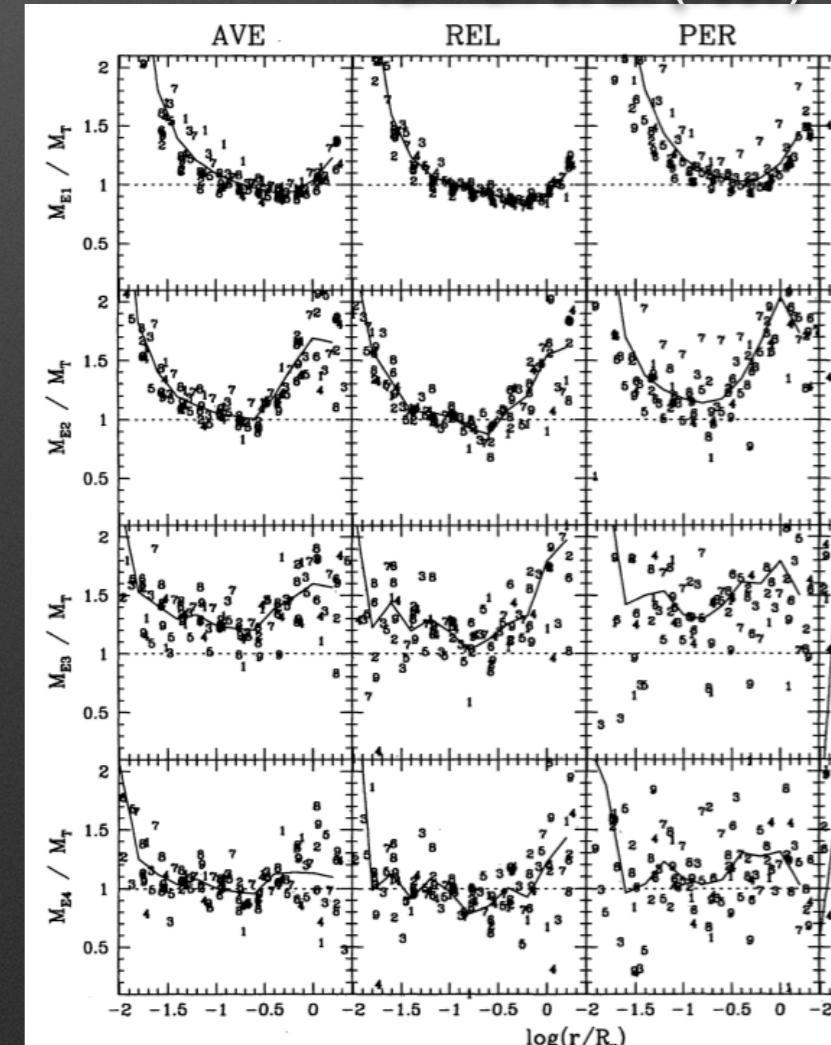
- many stellar systems look round -> spherically symmetric
- simplifying Jeans eq.
  - 2 unknowns: mass ( $v$ ) and anisotropy ( $\beta$ )
- good for triaxial CDM halos (Tormen et al. 1997) and stars in Es (e.g. Mamon et al. 2006)
- mass-anisotropy degeneracy (e.g. Binney & Mamon 1982, Mamon & Łokas 2005, Mamon & Boué 2010...)
  - assume mass profile
  - assume anisotropy profiles

stationary (non-streaming)  
spherical Jeans equation

$$\frac{d(\nu \sigma_r^2)}{dr} + 2 \frac{\beta}{r} \nu \sigma_r^2 = -\nu(r) \frac{v_c^2}{r}$$

$$\beta(r) = 1 - \frac{\sigma_\theta^2 + \sigma_\phi^2}{2 \sigma_r^2} = 1 - \frac{\sigma_\theta^2}{\sigma_r^2}$$

Tormen et al. (1997)



# Axisymmetric Jeans anisotropic models

- majority of galaxies are (nearly) axisymmetric (modulo bars and spiral arms)
- Jeans equations with some assumptions:
  - constant M/L ration
  - velocity ellipsoid aligned with cylindrical co-ordinate system, where  $\beta = 1 - \sigma_z^2 / \sigma_R^2$
  - approximate assumptions (e.g. Dehnen & Binney 1993, Binney 2014), but working (Lablanche et al. 2012)
- two free parameters  $\beta$  and **inclination** give a unique prediction of 2nd velocity moment:  $\langle V_{\text{los}}^2 \rangle$
- empirically,  $V_{\text{los}} = V_{\text{rms}} = \sqrt{V^2 + \sigma^2}$

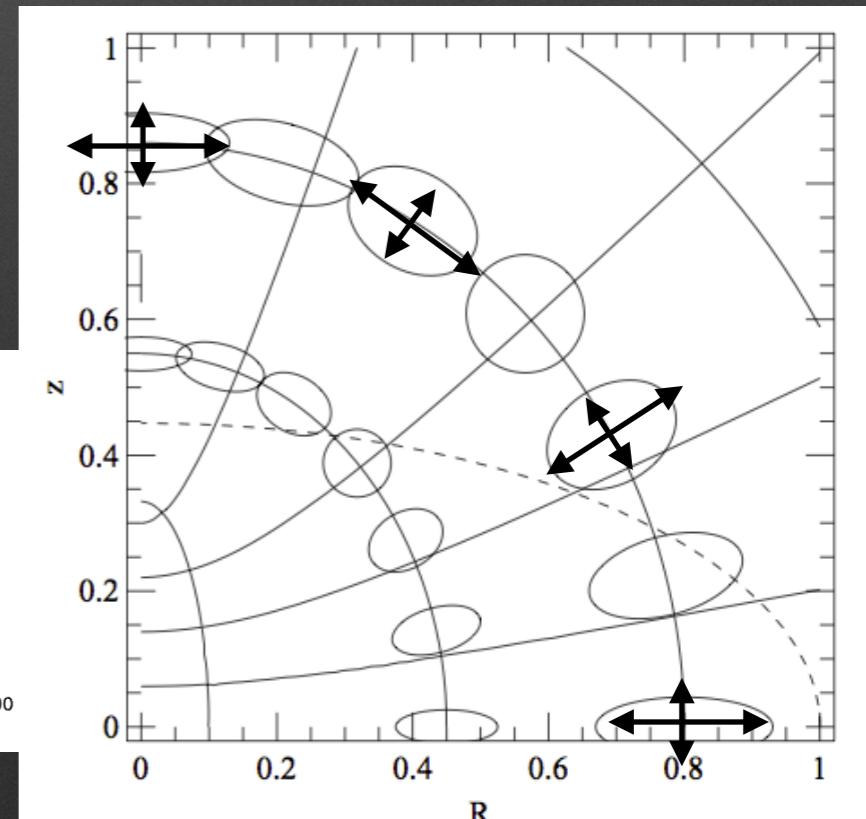
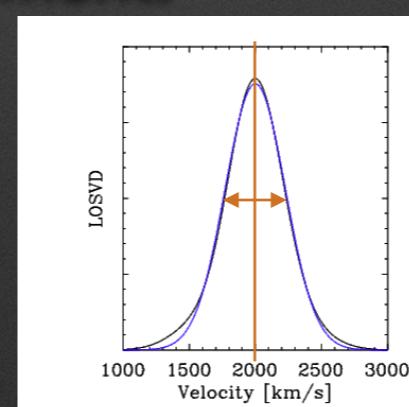
$$\frac{\nu\sigma_R^2 - \nu\bar{v}_\phi^2}{R} + \frac{\partial(\nu\sigma_R^2)}{\partial R} = -\nu\frac{\partial\Phi}{\partial R}$$

$$\frac{\partial(\nu\sigma_z^2)}{\partial z} = -\nu\frac{\partial\Phi}{\partial z}$$

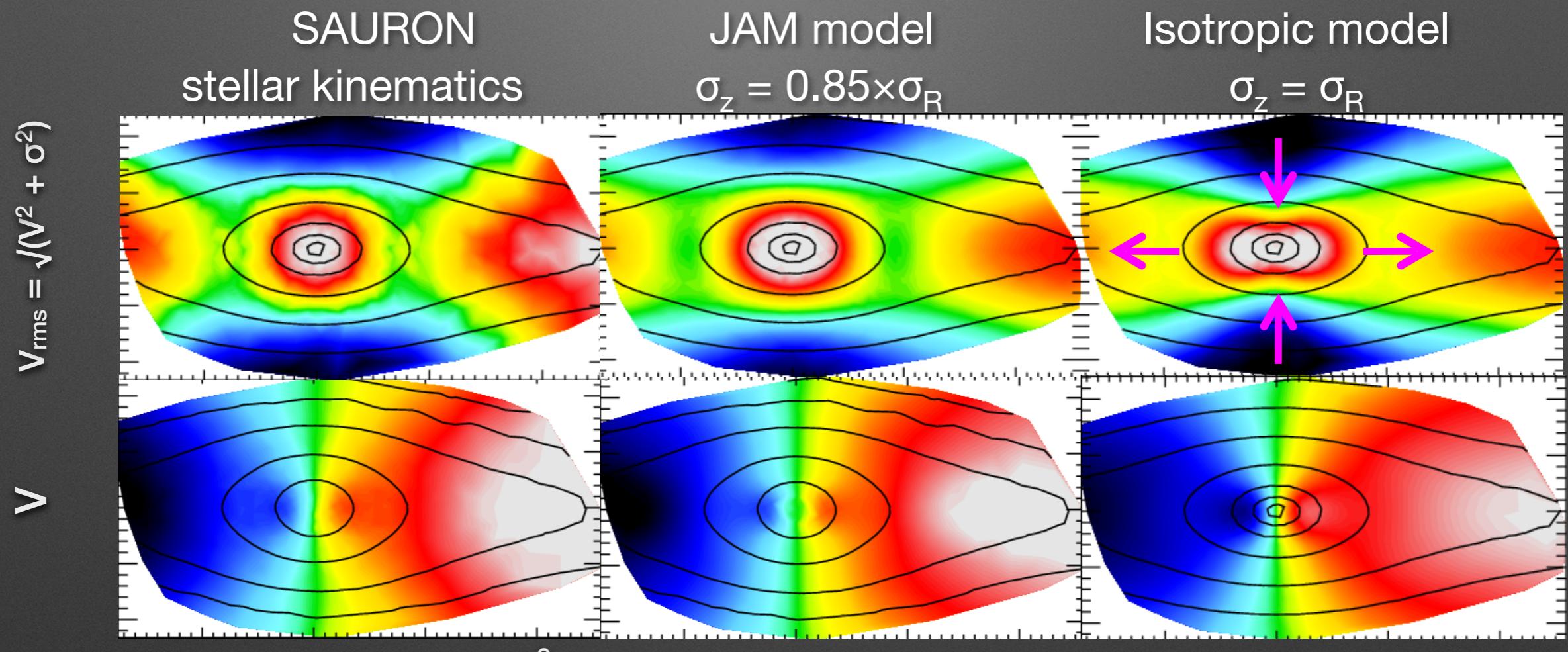
Cappellari et al. (2008)  $\sigma_R^2 = b\sigma_z^2$   
solutions:

$$\nu\sigma_z^2(R, z) = \int_z^\infty \nu\frac{\partial\Phi}{\partial z} dz$$

$$\nu\bar{v}_\phi^2(R, z) = b \left[ R\frac{\partial(\nu\sigma_z^2)}{\partial R} + \nu\sigma_z^2 \right] + R\nu\frac{\partial\Phi}{\partial R}$$



# JAM at work



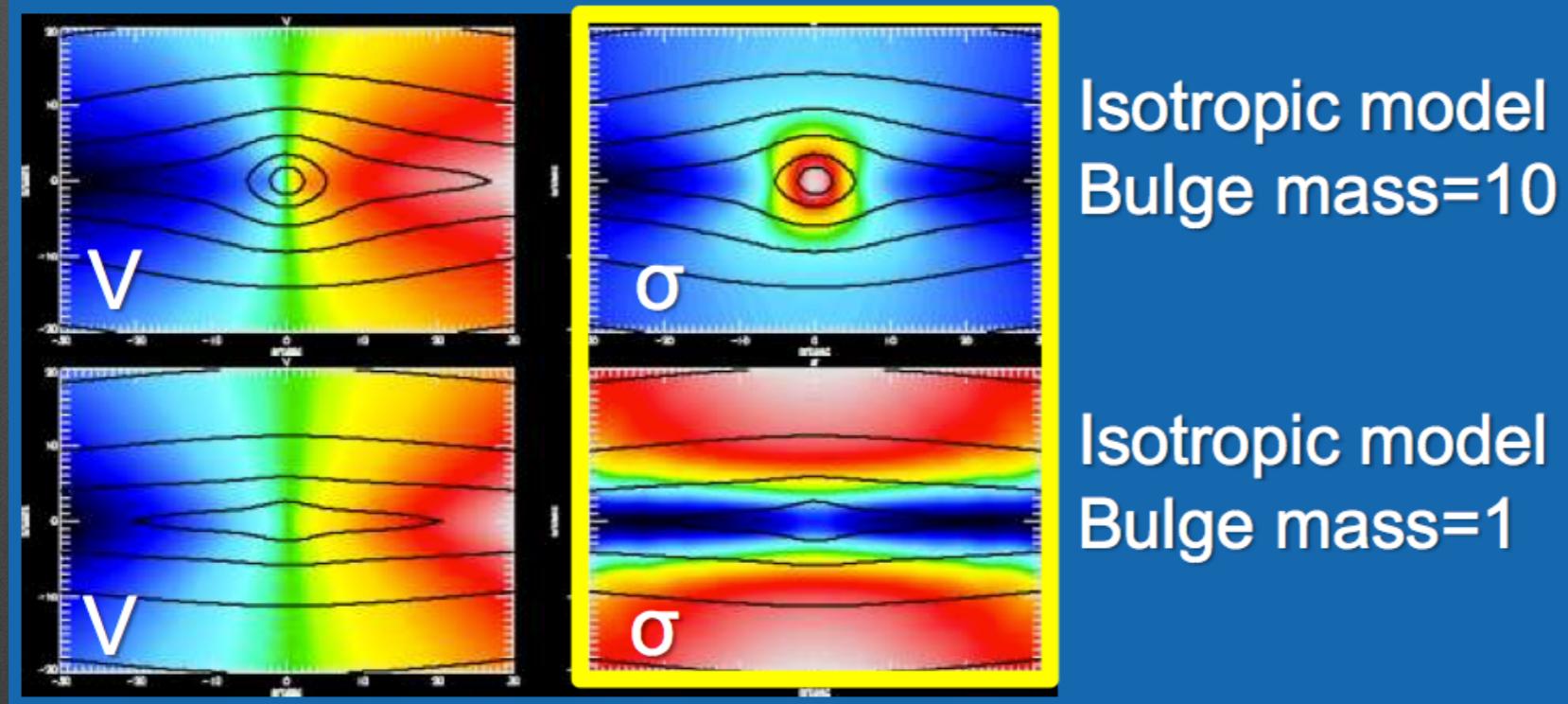
- Fast Rotators have  $\sigma_z < \sigma_R \approx \sigma_\phi^2$  (Cappellari et al. 2007, Thomas et al. 2009)
- ingredients: good light model (imaging) + good kinematics (IFU) (+add favourite dark matter model)
- efficient way to accurate M/L for a large samples of galaxies (+ Bayesian approach: Cappellari et al. 2013)

Slide: thanks to M. Cappellari

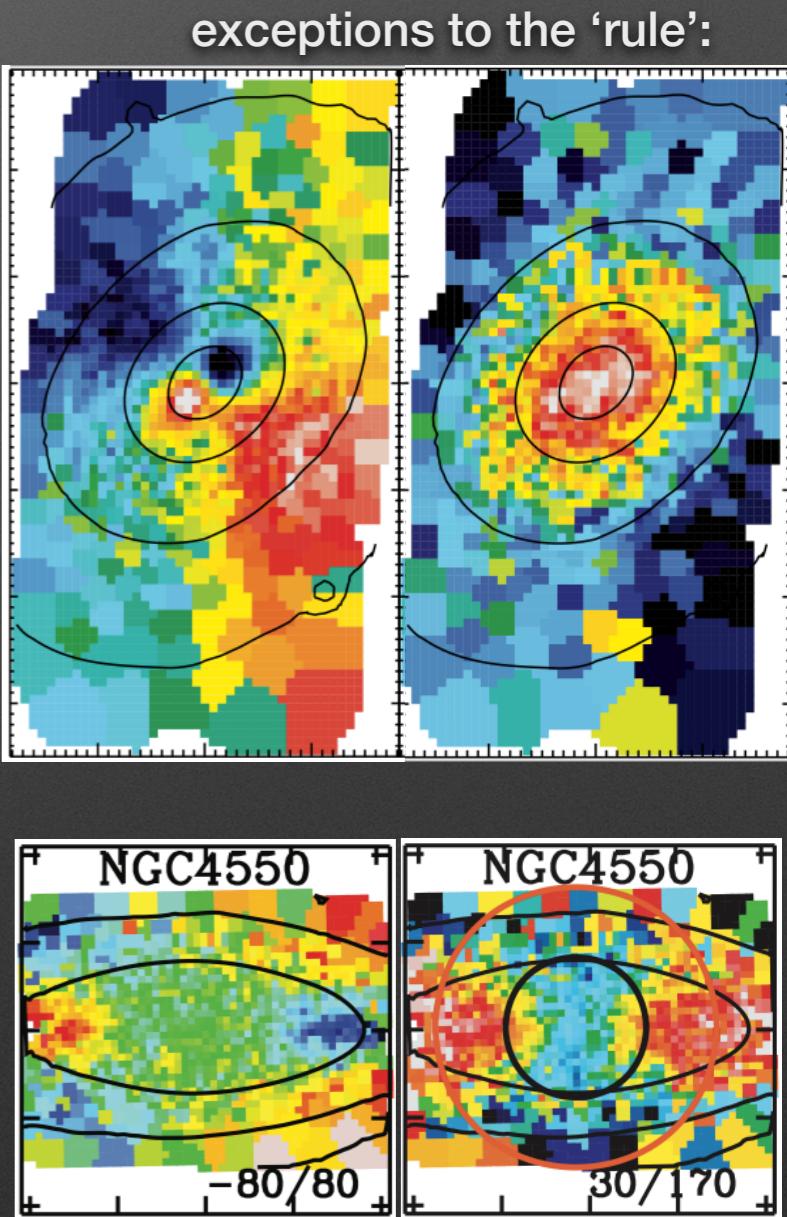
DAGAL 2015

# JAM prediction

from talk by M.Cappellari at “Fornax, Virgo, Coma et al.”, ESO, 2011



- Two model galaxies, one with massive and one with small bulge
- Differences encoded in photometry (e.g.  $\epsilon$ ) —> JAM predicts different kinematics
- 1)  $\sigma$  traces mass (not anisotropy!!)
- 2) anisotropy and kinematics are encoded in photometry!  $\sigma$  “normalises” (gives mass)
- exceptions: KDCs and counter rotating disks!



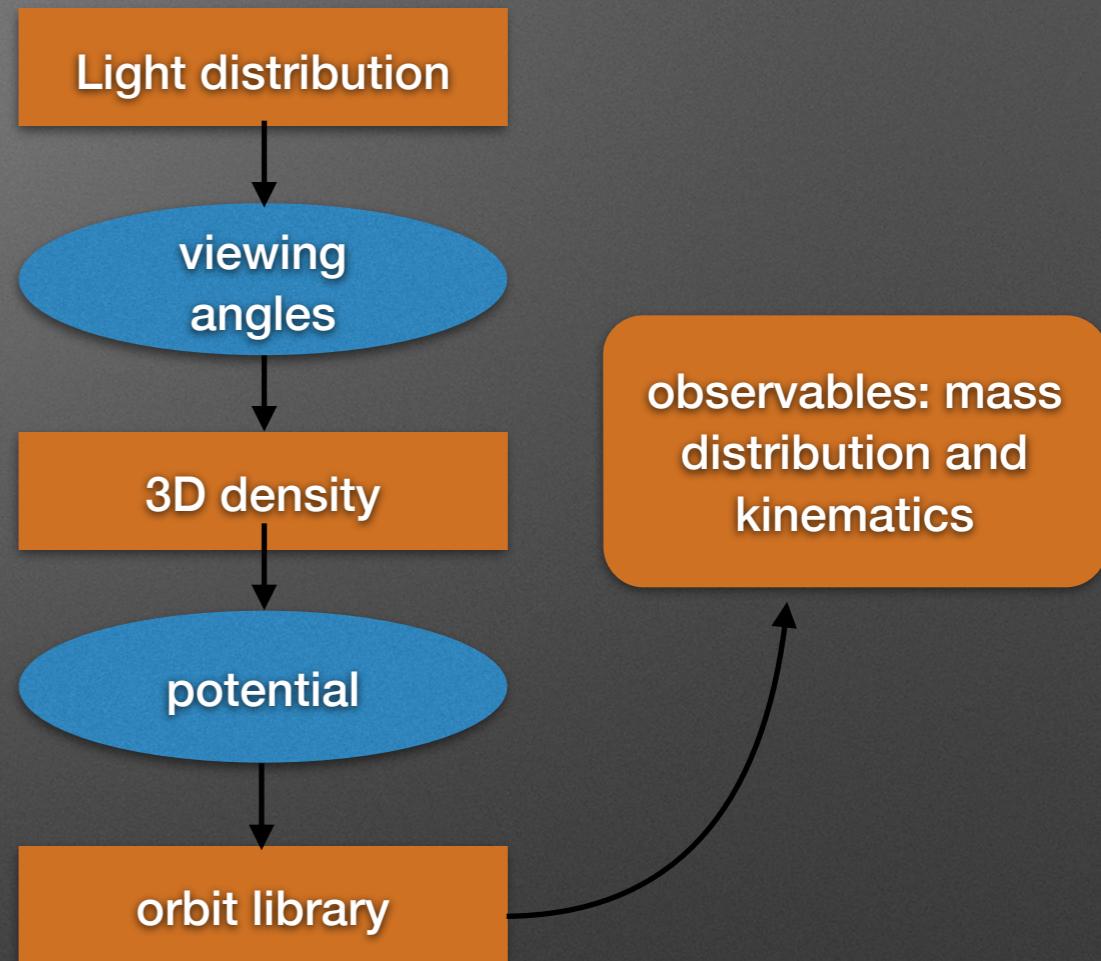
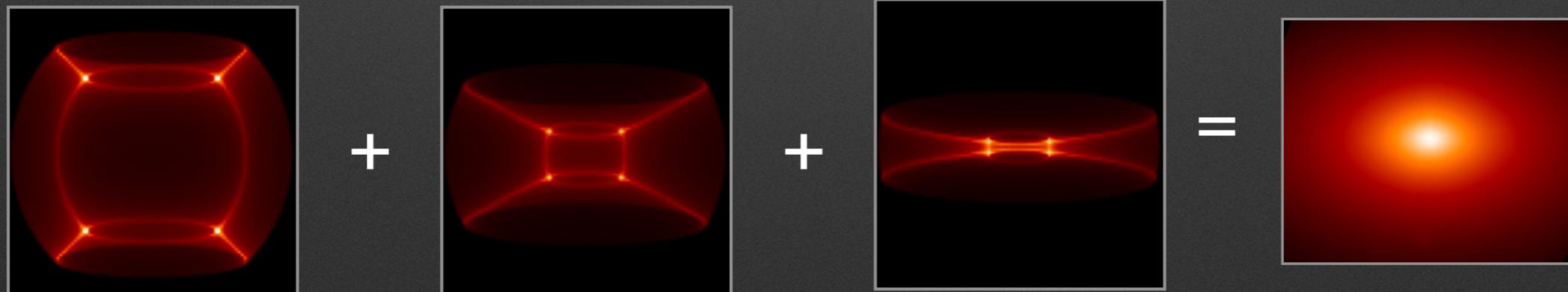
# DF via integrals of motion

- how to find the right distribution function  $f$ ?
- Jeans theorem helps
- $f(E)$ ,  $F(E, L_z)$ ,  $f(E, L_z, I_3)$
- guess a functional form for  $f$
- many different modelling approaches (symmetries, types of  $f$ )
- spherical distribution functions (e.g. Dejonghe & Merritt 1992, Mamon et al. 2013)
- flattened distribution functions (e.g. Evans 1994; Gerhard et al. 1998; Emsellem et al. 1999)
- “separable potentials” of the Stäckel from (e.g. de Zeeuw 1985, van de Ven et al. 2003)
- “numerical distribution functions” - general orbit based methods

**Jeans theorem (Jeans 1915):**  
 $f$  depends on the 6 phase-space coordinates  $(x, v)$  only through the isolating integral of motion (admitted by the gravitational potential of the system)  
(Jeans 1915, Lynden-Bell 1962, BT98,08)

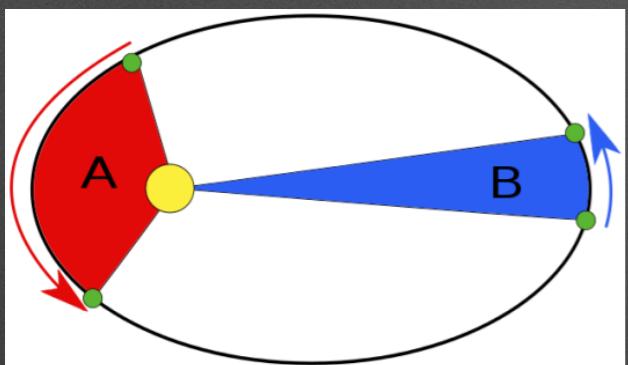
# Orbit based models (Schwarzschild's method)

- orbit superposition dynamical models (Schwarzschild 1979; Richstone & Tremaine 1984)
- creating a numerical representation of DF
- **axisymmetric** (e.g. Rix et al. 1997; van den Marel et al. 1998; Cretton et al. 1998; Gebhardt et al. 2003, Valuri et al. 2003, Cappellari et al. 2006; Thomas et al. 2009) and **triaxial** versions (van den Bosch et al. 2008, 2009a,b)
- very powerful, but some degeneracies exist (e.g. Krajnović et al. 2005, van den Bosch & van de Ven et al. 2009, de Lorenzi et al. 2009)

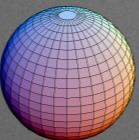


# Orbital structure

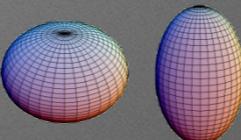
Point-like



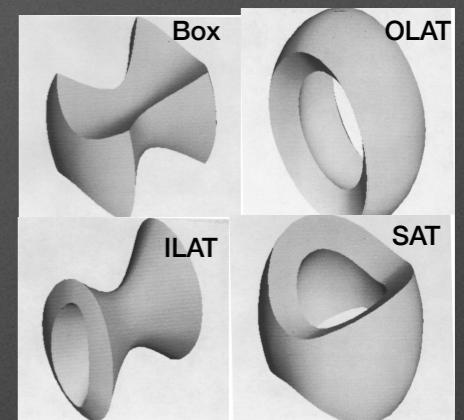
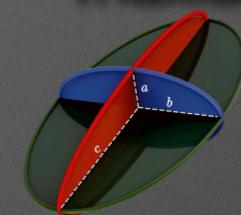
Spherical



Axisymmetric



Triaxial



Statler (1987)

Statler (1987)

- simple orbits in simple potentials
- axisymmetric: 1 major orbital family: short axis tubes (SAT)
- triaxial: 3 major orbital families: short (SAT) and long axis tubes (ILAT, OLAT) and box orbits (no angular momentum) (e.g. de Zeeuw 1984)

# Orbits of stars

- $f(E, L_z, I_3)$
- three integrals of motion:
  - energy, projection of angular momentum and “the third integral”
- $L_z \neq 0$ : orbits with sense of rotation, but streaming possible in both directions (prograde/retrograde)
- orbits don't fill the full area within the ZVC –
  - > the third integral of motions (Contopoulos 1960, Ollongren 1962, Martinet & Mayer 1975, Richstone 1982...)
- $I_3$  often not exact
  - exact in special cases: Stäckel potentials (e.g. de Zeeuw 1985)
- radius of the orbit : energy
- angular momentum  $L/L_{\max}$ ; both  $L < 0$  and  $L > 0$
- hight above the  $z=0$ :  $I_3$

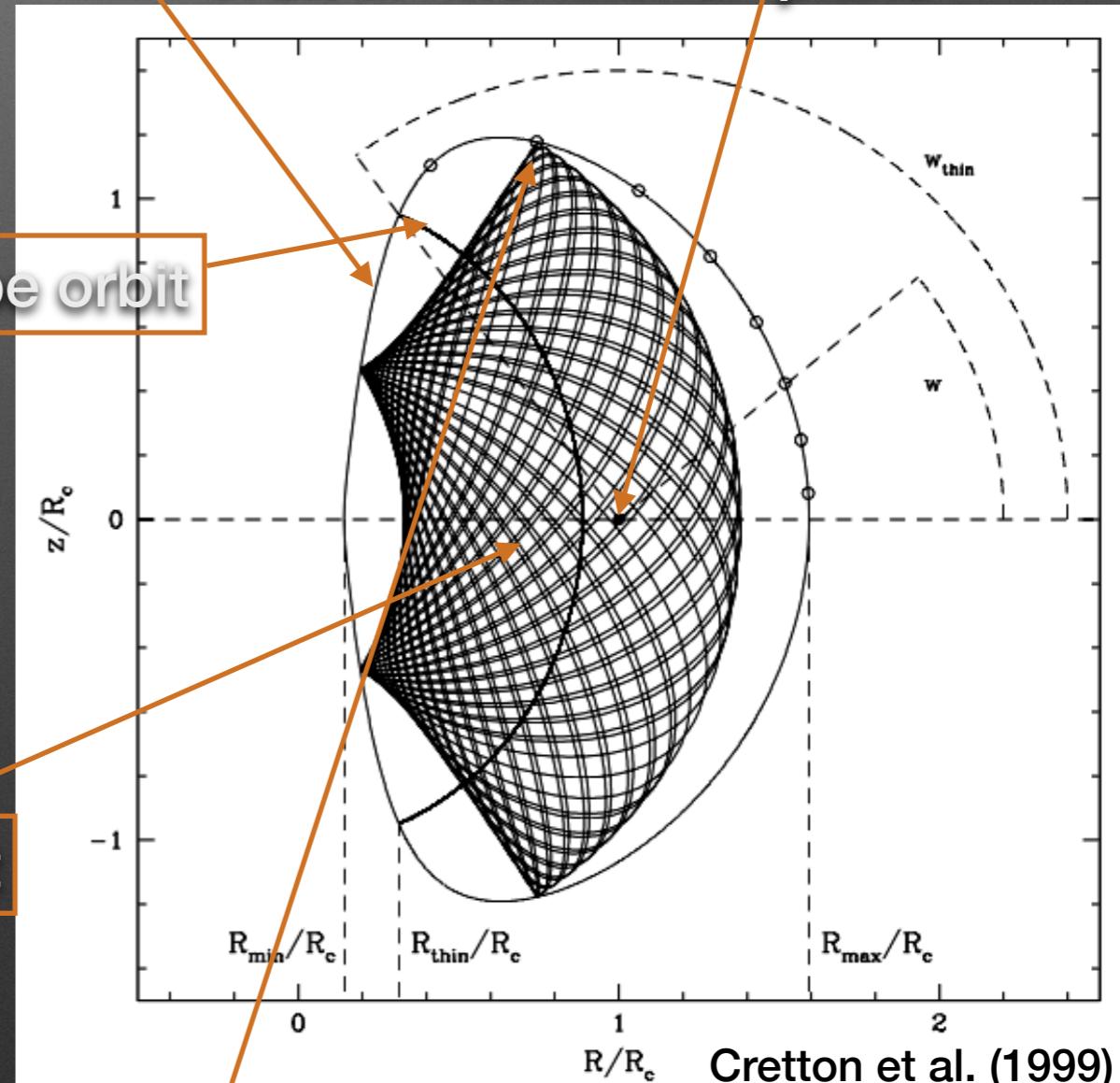
zero velocity curve (ZVC)

orbit in ‘meridional’ plane

thin tube orbit

regular orbit

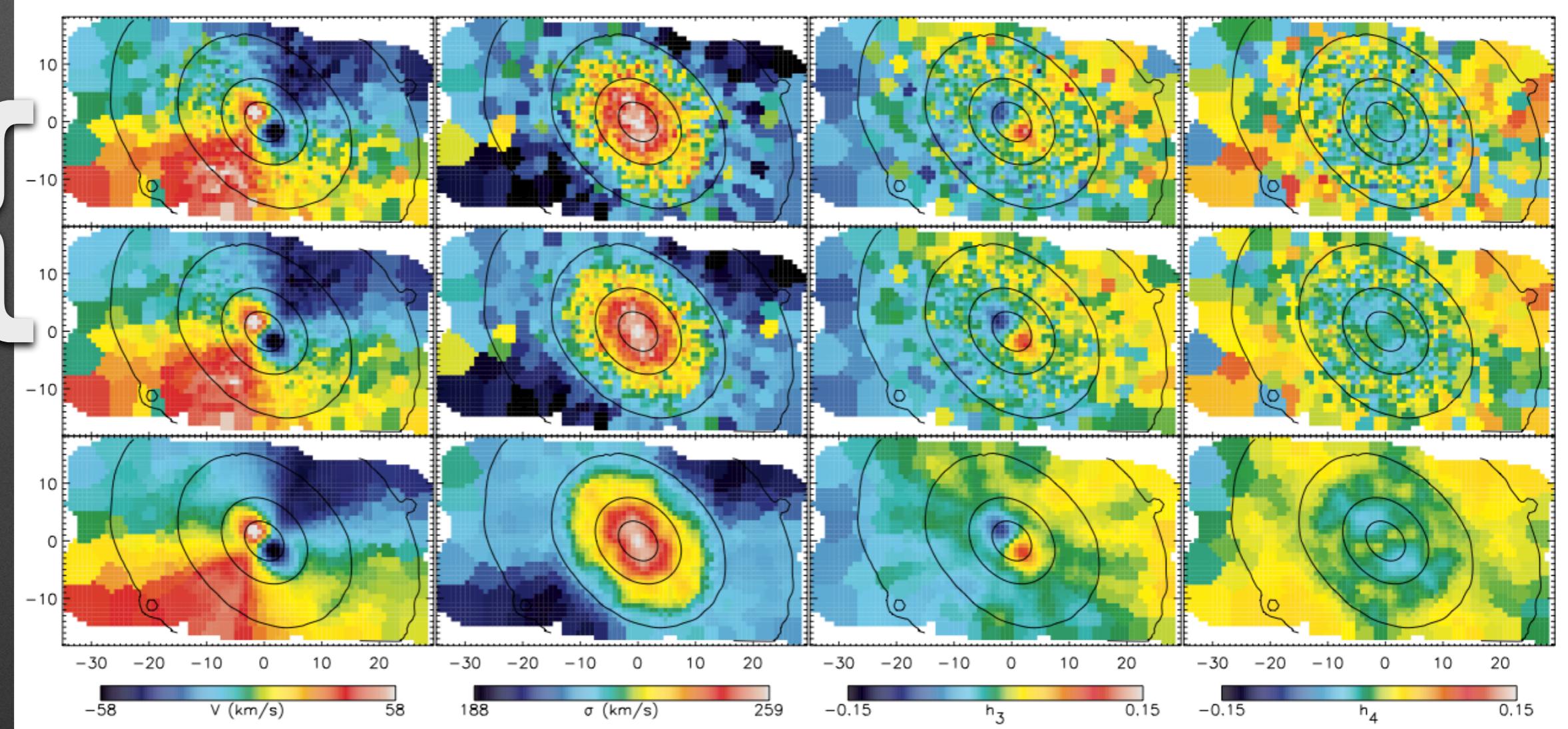
parameterising  $I_3$



circular orbit  
(max  $L$ )

# The power of Schwarzschild's method

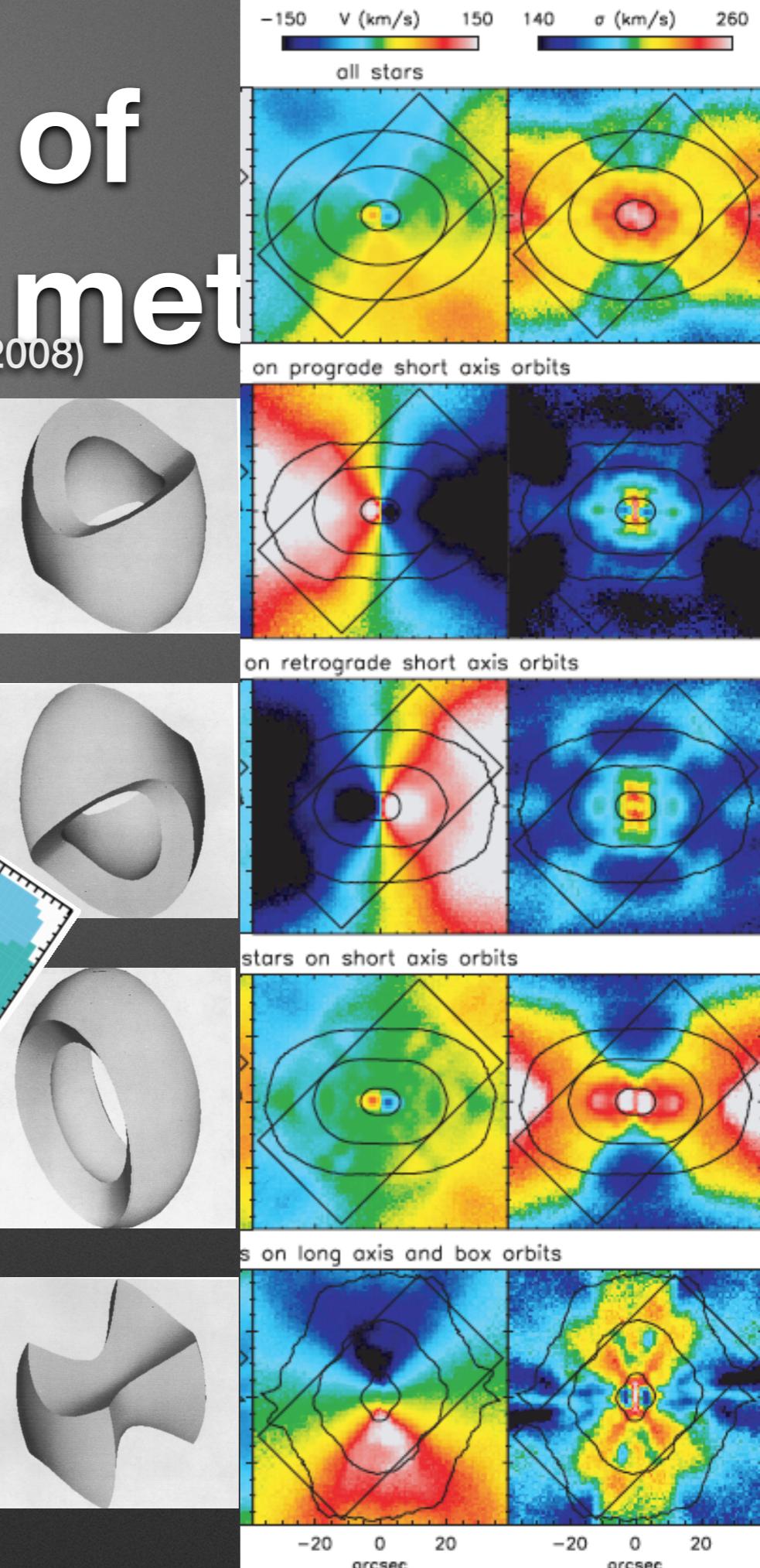
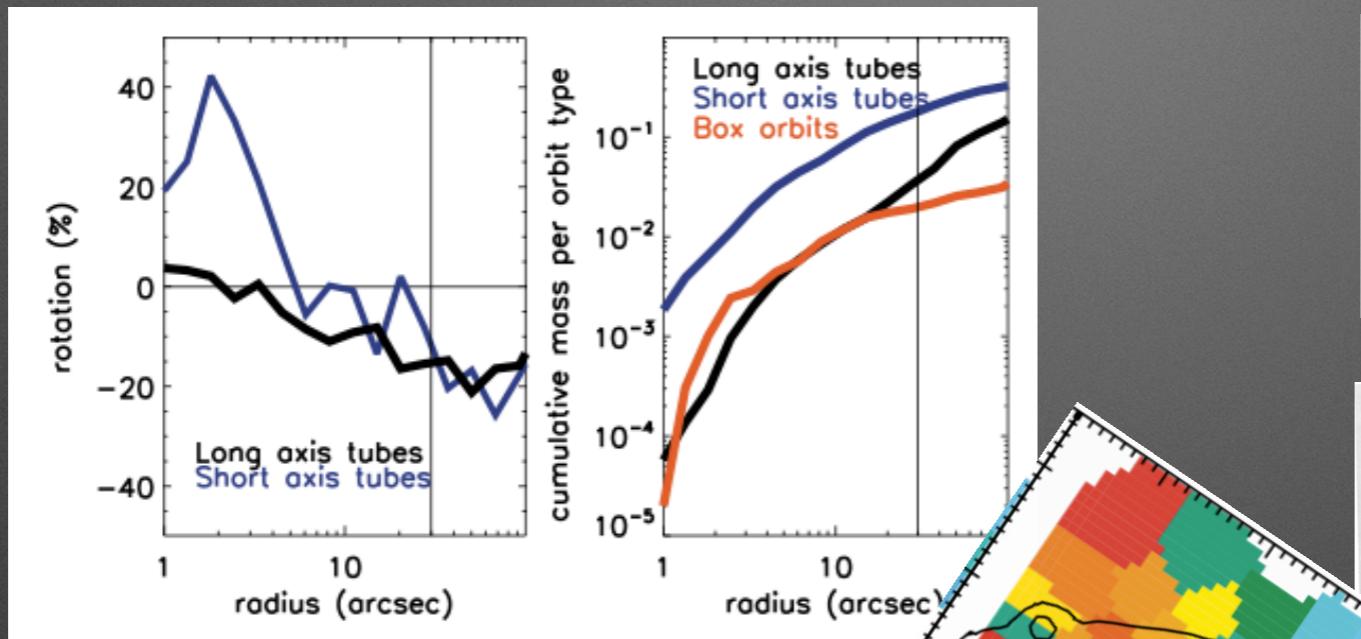
Data {



- triaxial Schwarzschild models (van den Bosch et al. 2009)
- include short-axis tubes, long-axis tubed and box orbits

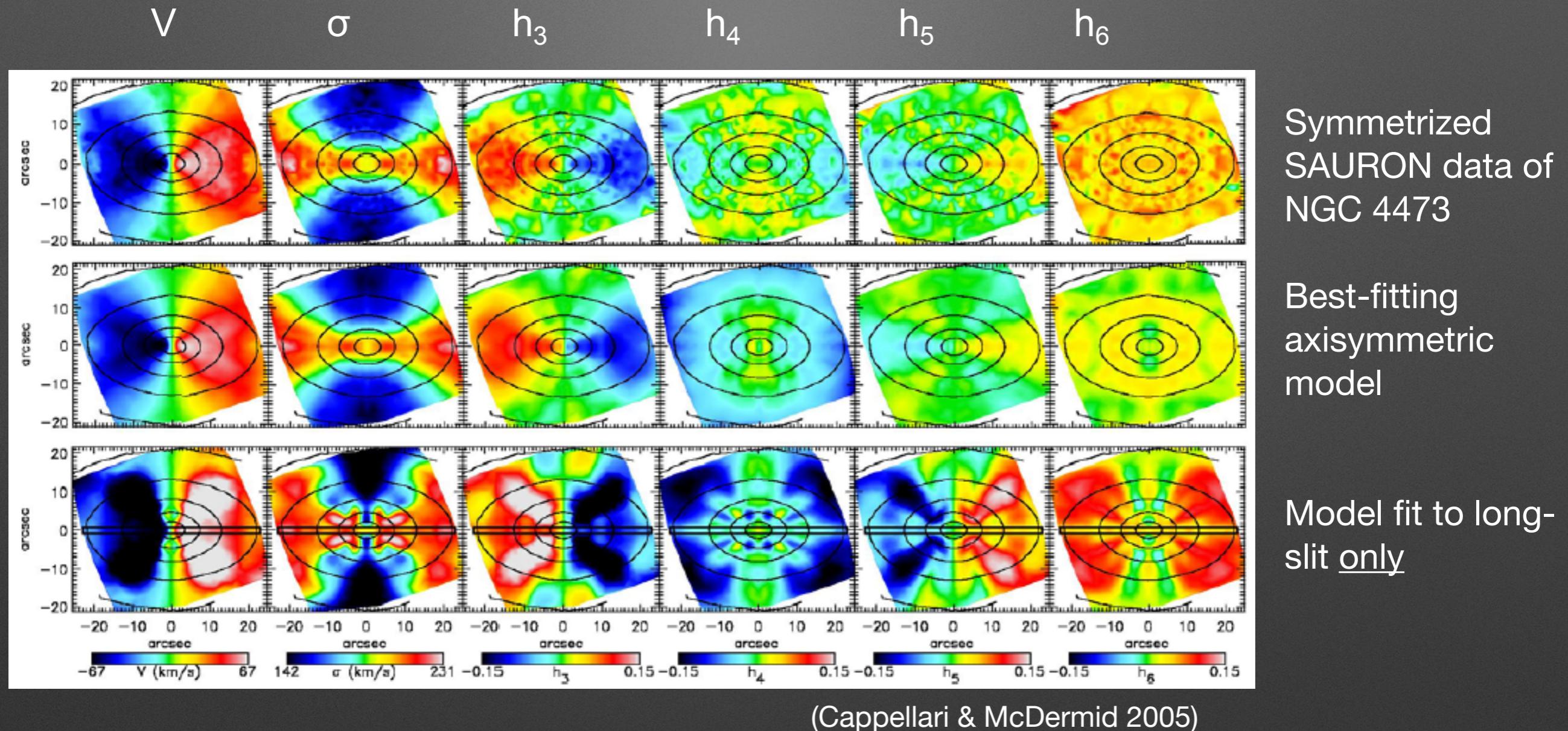
# The power of Schwarzschild's method

van den Bosch et al. (2008)



- several orbital families are present in the body of the galaxy
- no change in type between KDC and KDC+1
- change in mass fraction of
- KDC is the ‘orbital-composite’ solution
- dynamically stable structure

# The need for integral-field coverage

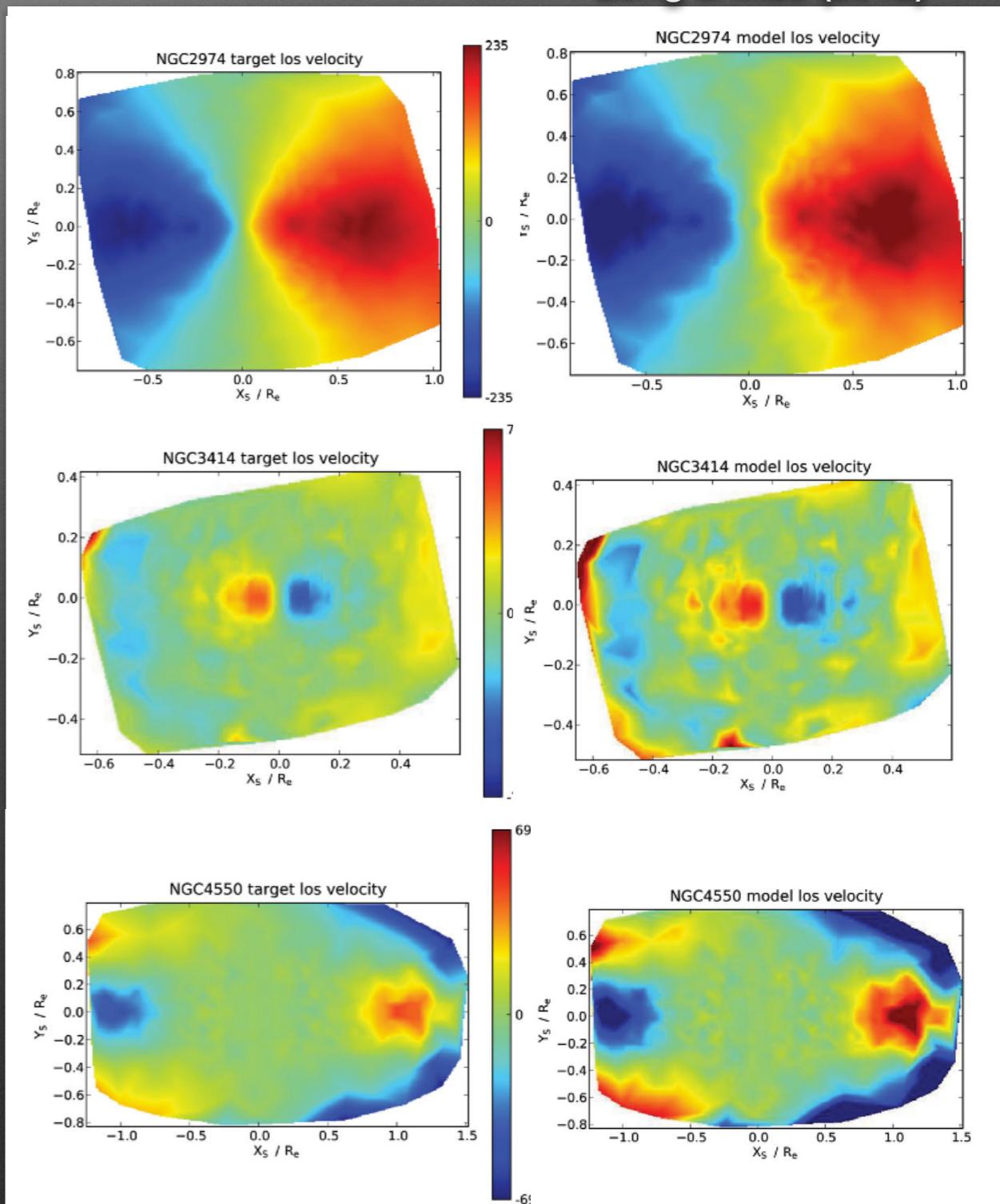


- Dimensional argument: DF is 3D → need 3D data
- Little can be recovered of the true galaxy dynamics from single long-slit data

# Made to Measure Models

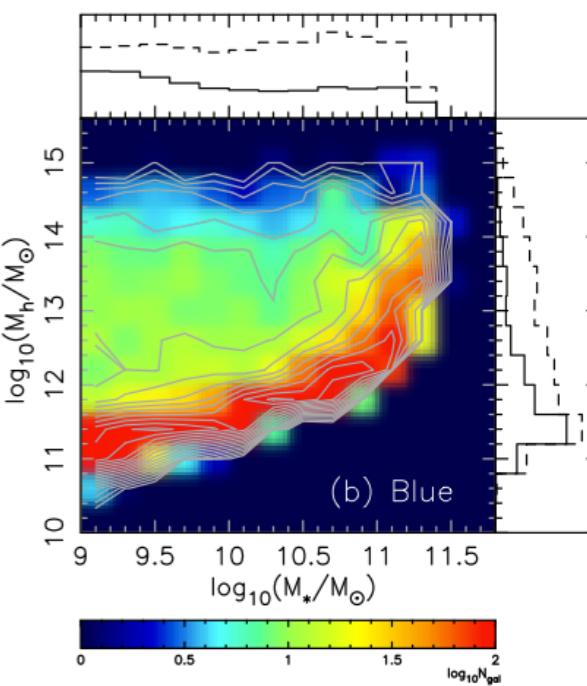
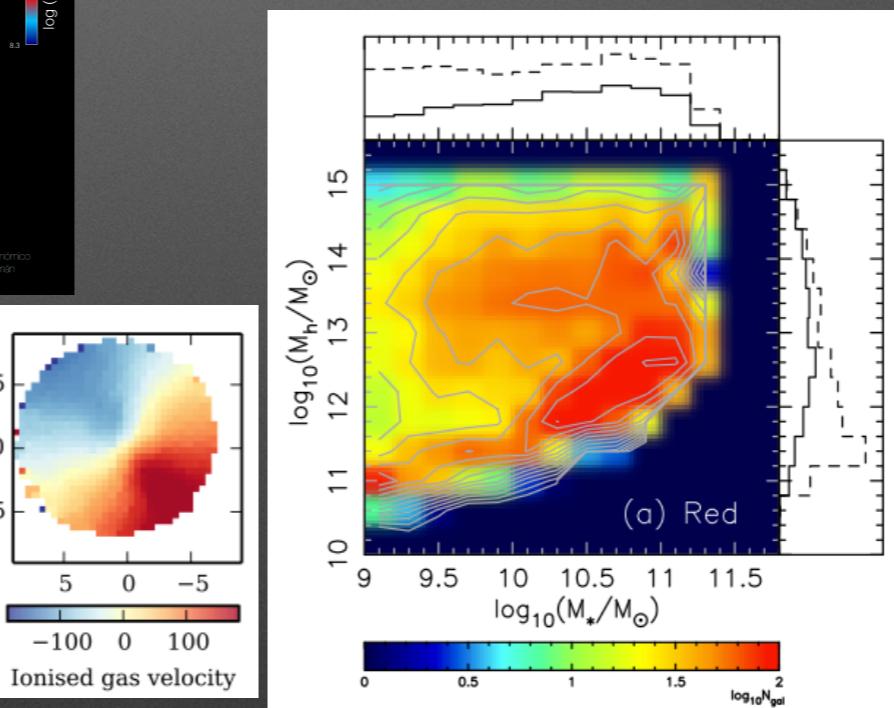
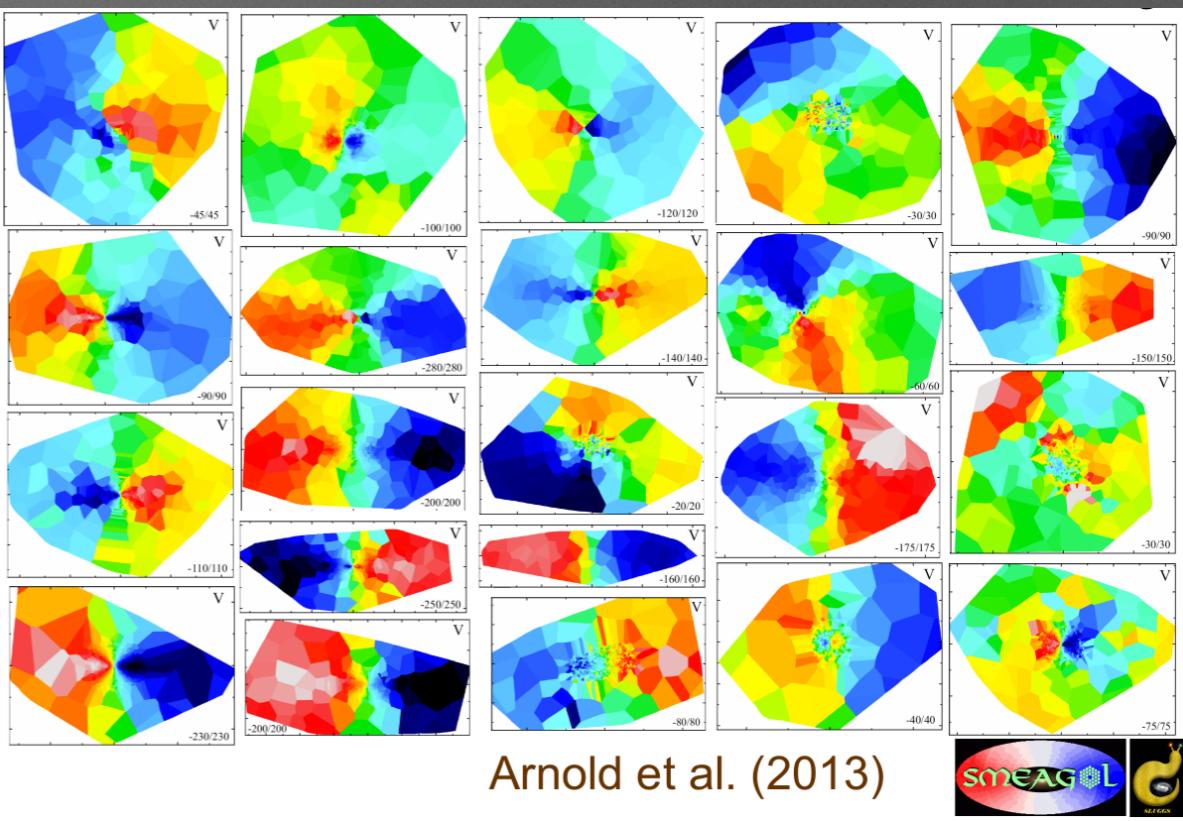
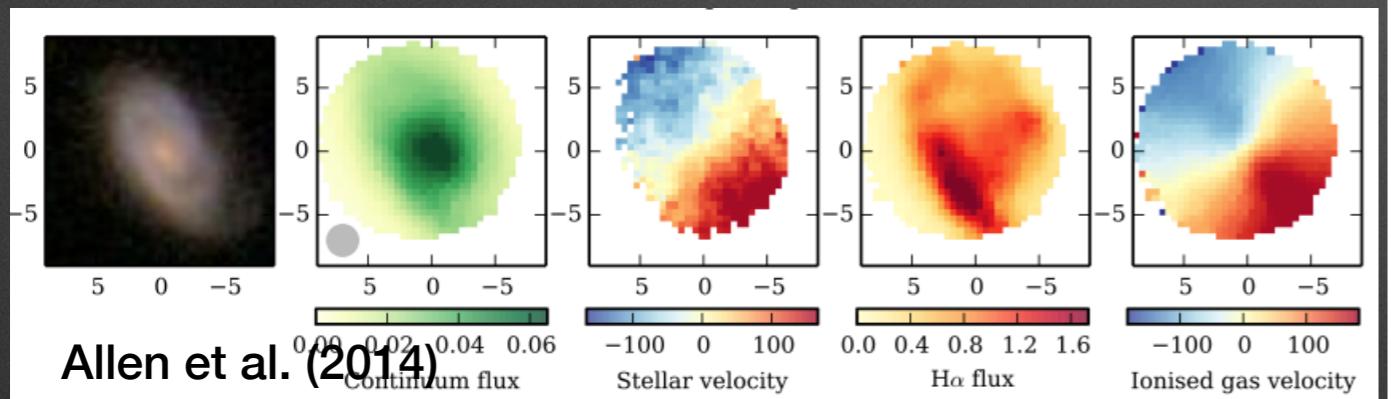
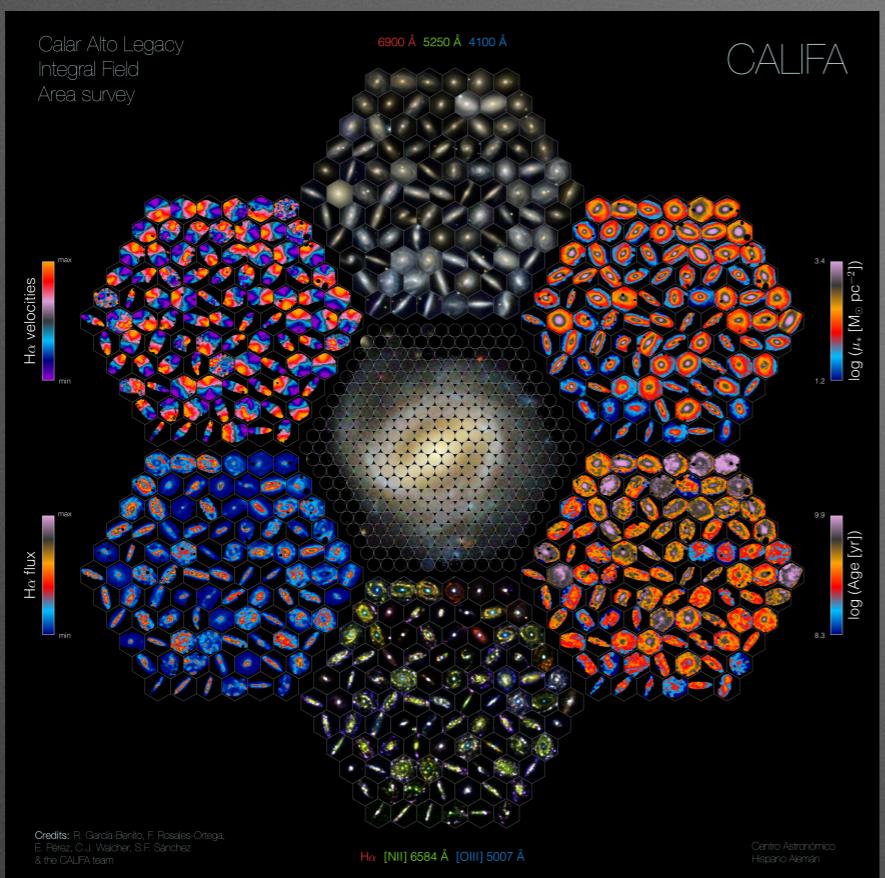
- Schwarzschild's method averages observable over an orbit (1st make orbit, then weight it)
- observable can also be continuously updated (weights determined dynamically as particle orbits)
- a mixture of N-body and Schwarzschild's method: system of N particles and N weights ( $w$ )
- Syer & Termaine (1996) method or Made-2-Measure (de Lorenzi et al. 2007, Jourdeuil & Emsellem 2007, Long & Mao 2010)
- less well developed, but powerful (Long & Mao 2012, 2013; Das et al. 2011; Morganti et al. 2013)

Long & Mao (2012)



# **Outlook: the future is IFU**

# More surveys with IFUs



Comparison of IFU Surveys

Bundy et al. (2015)

Specification	MaNGA	SAMI	CALIFA	DiskMass (H $\alpha$ )	DiskMass (Stellar)	ATLAS <sup>3D</sup>
Sample size	10,000	3400	600	146	46	260
Selection	$M_* > 10^9 M_\odot$	$M_* > 10^{8.2} M_\odot$	$45'' < D_{25} < 80''$	S/SAab-cd, b/a > 0.75 $10'' < h_R < 20''$	$M_* \gtrsim 10^{9.8} M_\odot$ E/S0	