The Hubble sequence of galaxies

Problem set 2

Reading: the relevant material is in Sections 5 and 6 of Sparke & Gallagher (SG): "Galaxies in the Universe: An Introduction" (edition 2000) and in Sections 4.1-4.4 of Binney and Merrifield "Galactic Astronomy" (1998). The virial theorem is addressed in Section 4.8.3 of Binney and Tremaine and Sections 3.2 and 6.2.3 of SG. Note that the total number of points is 120, because problem 3c), with 20 extra points, is not required. It is given for fun.

1. **[10 points]** The Hubble Sequence of galaxies.

   - Describe the types of galaxies on the Hubble sequence.
   - What is the purpose of morphological classifications like the Hubble sequence?
   - What are the problems with the Hubble classification?

2. **[30 points]** Surface brightness profiles of galaxies. Neglecting features like spiral arms (and bars), surface brightness profiles of stellar disk galaxies can be well described by the following expression:

$$I(R) = I_0 \exp[-\frac{R}{R_s}]$$

where I_0 is the *central surface brightness* and R_s is the *scale length*. Surface brightness profiles of spheroidal components, such as bulges of disk galaxies and elliptical galaxies, are well described by an empirical function presented by Sérsic (1968):

$$I(R) = I_0 \exp[-(\frac{R}{R_0})^{1/n}]$$

where I_0 is the central surface brightness and R_0 a characteristic radius. The parameter n can take any value, but $n = 0.5$ gives a Gaussian, $n = 1$ is equivalent to the exponential disk surface brightness profile and $n = 4$ is known as de Vaucouleurs profile, which describes relatively well elliptical galaxies.

While disk parameters are typically expressed in terms of the scale length and the central surface brightness (R_s and I_0), it is customary to express parameters or early-type galaxies in terms of the *effective radius* and *effective surface brightness* (R_e and I_e), where effective radius encloses half the total extrapolated luminosity. In this notation Sérsic function is given by

$$I(R) = I_e \exp\{-b_n[(\frac{R}{R_e})^{1/n} - 1]\}$$

where b_n is chosen to ensure that

$$\int_0^\infty I_b(r) 2\pi r dr = 2\int_0^{R_e} I_b(r) 2\pi r dr$$

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Since this equation can not be solved analytically, we use numerical approximations. Ciotti & Bertin (1999) give the following excellent approximation:

$$b_n = 2n - \frac{1}{3} + \frac{4}{405n} + \frac{46}{25515n^2}$$

a) Show that the total disk luminosity is given by $L_{tot} = 2\pi I_0 R_s^2$.

b) Derive the value for R_e of a pure exponential disk in terms of its scale hight R_s.

c) What is the total luminosity for a de Vaucouleurs profile ($n = 4$)?

d) What is the total luminosity for a general Sérsic function? Using expressions derived in 2a) and 2d) derive the expression for bulge-to-disc ratio. **Hint**: make use of a Γ function.

e) For both the exponential disk and the general Sérsic function give equivalent expressions using surface brightness in magnitudes (μ_e).

f) Plot the Sérsic function for n=[0.2, 0.6, 1, 2, 3, 4, 6, 8, 10] (use μ_e=18 mag/arcsec^2 and R_e=5 arcsec. Comment on the properties of these functions. What sort of objects would have a small and what a large n?

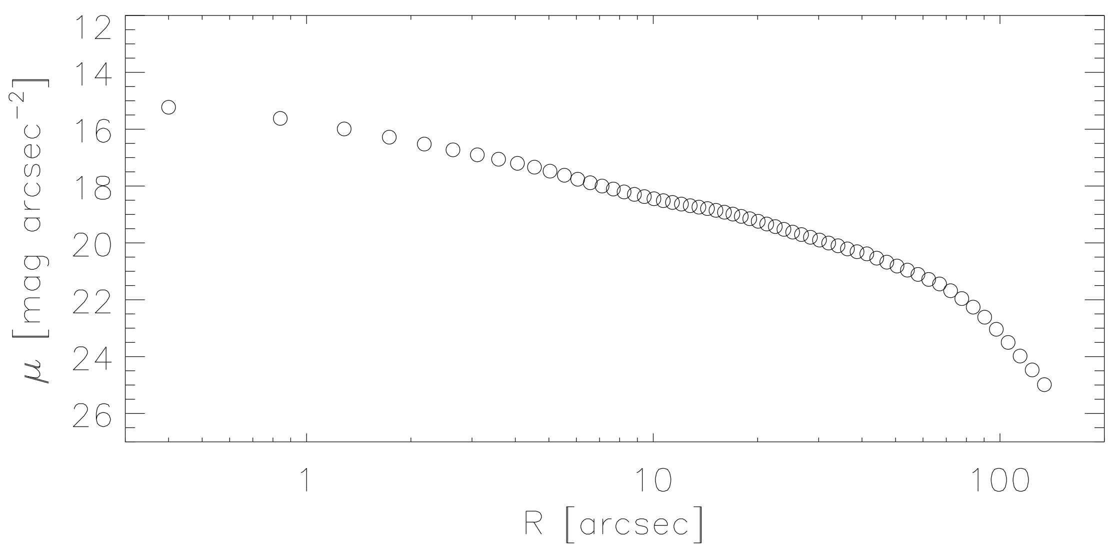

Figure 1: Surface brightness profile of a galaxy, which possibly has two components: a large disk and a small bulge. Can you see them? (Uncertainties are typically of the order of 0.1 mag, hence smaller than the symbols.) Data to generate this plot are in the file galaxy_profile.txt

3. **[20 points (+ 20 extra points)]** Disk–bulge decomposition is very popular in modern extragalactic astronomy. The idea is that an object consists of two components, a disk and a bulge, and using the Sérsic function one tries to separate these two components. The problem is highly degenerate and strongly dependent on the viewing angle (i.e. it is not certain what one recovers is really true), but it can be done. On Fig. 1 there is a light profile of an early-type galaxy (data are in the file galaxy_profile.txt).

a) Fit a single de Vaucouleurs function to the profile shown. What are the parameters of the fit? **Hint**: it might be easier to fit the profile while it is in counts (also in the file) and then convert everything into surface brightness. Look for solutions only between about 2.5 and 130 arcsec.

b) Fit a single Sérsic function (free n) to the profile (use the same range as above). What are the parameters of the best fit function? How does it compare with the de Vaucouleurs profile fit?

c) **For extra 20 points.** Fit a combination of a general Sérsic function (free n) and an pure exponential profile ($n = 1$) to the given profile. What do you obtain as the B/D in this case? How big is the bulge and the disk of this object? **Hint**: you should fit a function of this kind: $I = (I_{bulge} + I_{disk})$, where $I_{bulge} = I_e \exp\{-b_n[(\frac{R}{R_e})^{1/n} - 1]\}$ is the general Sérsic function, $I_{disk} = I_0 \exp[-\frac{R}{R_s}]$ is the exponential profile. Look for solutions only between about 2.5 and 130 arcsec. When calculating the total luminosity of the bulge and disc, assume that the flattening of the bulge q_b=0.65 and the flattening of the disc q_d=0.35 (you will have to modify the integrals from 2a) and 2d) problems to account for the flattening).

4. **[15 points]** Virial Theorem and Mass – Size diagram

a) Using the virial theorem derive the expression for the virial mass of a stellar system. When dealing with observed galaxies one typically uses effective (half-light) radius and velocity dispersion. Look in Cappellari et al. (2006, MNRAS, 366, 1126) for the calibrated expression for virial mass of early-type galaxies. Comment on its value (is it expected?) and how robust is this result? What does the scale factor depend on?Why is it useful?

b) Use Falcón-Barroso et al. (2011, MNRAS, 417, 1787) data to construct mass – size diagram for early-type (E/S0) and Sa galaxies and over-plot lines of constant velocity dispersions for σ =50,70,100,130,170 and 240.. **Hint**: sizes and velocity dispersions are given in the paper (Tables D1-4), but you will have to make use of the Virial Theorem and the derived expression for the virial mass to estimate galaxy masses. Note that you need to convert the sizes to physical units (in kpc) and use the appropriate units for the gravitational constant. Plot in log-log.

c) What are the locations of Sa and E/S0 galaxies in this diagram? Is there are systematic difference and in what parameter(s) it is best recognised? What could be the origin of the trend?

5. **[10 points]** Scaling relations.

a) Describe the Tully-Fisher relation.

b) Describe the Faber-Jackson relation.

c) Outline why for both Tully-Fisher and Faber-Jacskon relations $L \sim V^4$ and $L \sim \sigma^4$. What is the assumption?

d) Why are these relations similar and what are their consequences?

6. **[15 points]** The Fundamental Plane.

a) Outline the properties of the Fundamental plane? What does the Fundamental plane imply?

b) What is the origin of the Fundamental plane? Assume that galaxies can be described by a single value of σ and M/L, no dark matter is present, and all galaxies follow $n = 4$ Sérsic law (use the solution of 2c). Derive the fundamental plane in form of $\log R_e = \alpha \,\mathrm{Log}\, \sigma + \beta\mu_e + \gamma$, where $\mu_e = -2.5log(I_e)$ is the surface brightness in mag/arcsec^2, R_e is the effective radius in kpc and σ is the velocity dispersion in km/s mad γ is a constant (i.e. find α and β).

c) Hyde & Bernardi (2009, MNRAS, 396,1171) give the following values in $r-$band: $\alpha = 1.43$, $\beta = 0.31$ and $\gamma = -8.9$. Compare these values with "theoretical" ones from *b*). How big is the difference between the theoretical and observed Fundamental Plane and discuss the meaning of the difference for galaxy structure.